

$$k = \frac{50}{10} = 0.2$$

$$C_s^* = k C_0 (1 - f_s)^{k-1}$$

First solid form $C_s^* = (0.2)(5)(1-0)^{0.2-1} = 1\%B$

$$10\% = (0.2)(5) f^{0.2-1}$$

$$10 = f^{-0.8}$$

$$f^e = 10^{-1.25} = 0.056$$

5pts

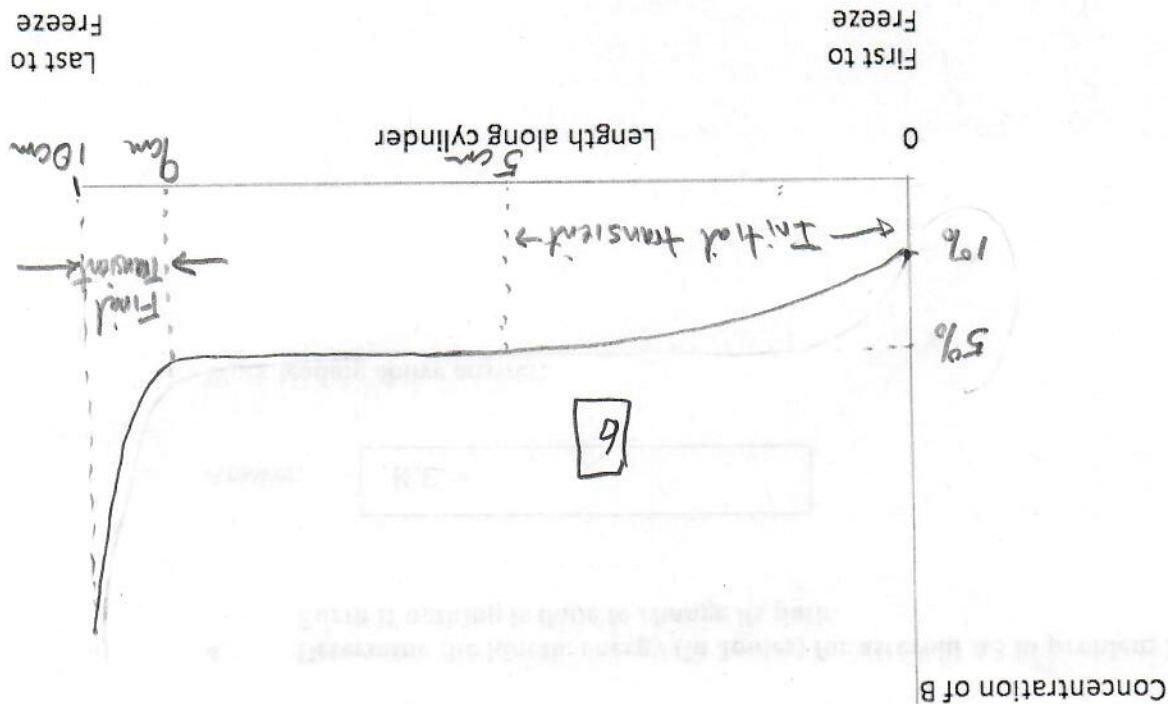
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length of final transient $\boxed{7}$

$$= \frac{D_c}{R} = \frac{1 \times 10^{-5} \text{ cm}^2/\text{s}}{0.1 \times 10^{-4} \text{ cm/s}} = 1 \text{ cm}$$

length of initial transient $\boxed{7}$

$$= \frac{D_c}{kR} = \frac{1 \times 10^{-5} \text{ cm}^2/\text{s}}{(0.2)(0.1 \times 10^{-4} \text{ cm/s})} = 5 \text{ cm}$$



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(b) For the ingot that solidified with infinite mixing in the liquid:
 (i) What fraction of the ingot has a concentration of B that is greater than 5%?

$$f_B \text{ in Eutectic} = \frac{90-50}{90-10} = 0.50$$

$$f^E = 0.056 \quad (5)$$

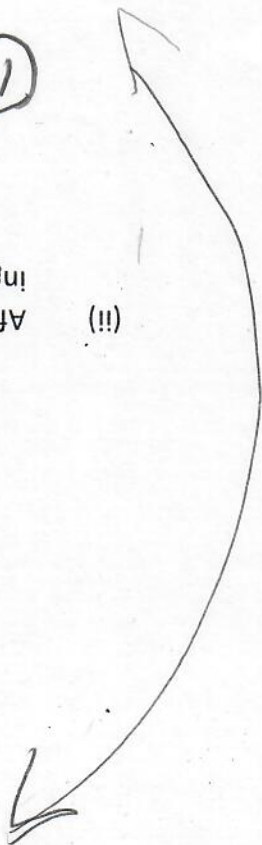
$$f_{0.05} \text{ in the ingot} = (0.50)(0.056) = 0.028$$

(ii) After solidification is complete what is the amount of β -phase in the ingot? (10)

$$5\% = (0.2)(5\%) f_{-0.8}^L$$

$$5 = f_{-0.8}^L$$

$$f_L^L = 5 - 1.25 = 0.1337$$



(c) A third ingot of the same dimensions and composition is frozen at the rate of 10 micron/s and with a thermal gradient in the liquid phase at the liquid-solid interface of 100°C/cm. Will solidification occur with a plane front or will the liquid-solid interface break down resulting in cellular or dendritic solidification? Explain your answer.

Criterion for PFS:

$$\frac{G_L}{R} \geq m_L C_s^* (1-k) / k \Delta T$$

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$$m_L = \frac{0 - 50\%}{1000^\circ\text{C} - 500^\circ\text{C}} = -10^\circ\text{C}/\% \quad \boxed{5}$$

A + steady state

$$C_s^* = C_0$$

$$\frac{G_L}{R} \geq \frac{m_L C_0 (1-k)}{k \Delta T} = \frac{(-10^\circ\text{C}/\%)(5\%)}{1 \times 10^{-5} \text{cm}^2/\text{s}} \left(\frac{1-0.2}{0.2} \right)$$

$$= \frac{200^\circ\text{C} \cdot 5/\text{cm}^2}{1 \times 10^{-5}}$$

$$\text{For } R = 10 \mu\text{m}/\text{s}$$

$$G_L \geq 2 \times 10^4 \text{ } ^\circ\text{C}/\text{cm} \quad \boxed{8}$$

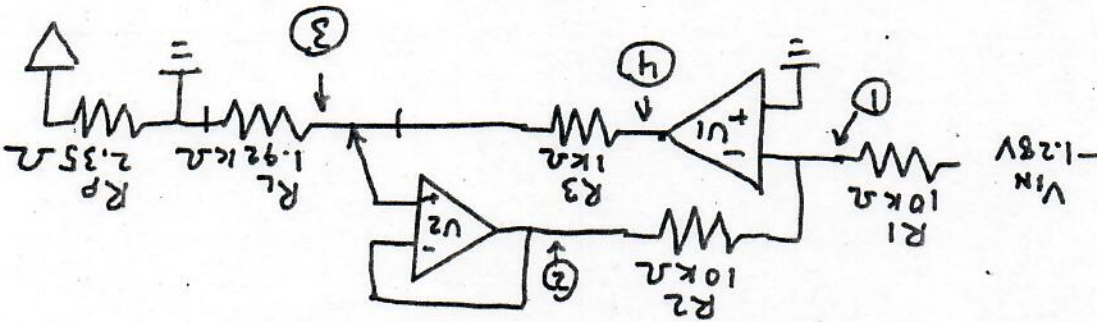
Actual gradient is 100°C

⇒ Plane front will breakdown

resulting in cellular or dendritic solidification

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2. Consider the following circuit.



- Determine the current at each of the four circled locations.
- Determine the voltage at each of the four circled locations.

Please state any assumptions that you make.

(1) Current into inputs of op amp = 0 3 pts.

(2) With negative feedback $V^+ = V^-$ 3 pts.

① $V^- = V^+ = 0$

$\Rightarrow I_{at\ 1} = \frac{1.28V}{10k\Omega} = 1.28 \times 10^{-4} A$

② $I_{through\ R2} = 1.28 \times 10^{-4} A \Rightarrow V_{at\ 2} = (1.28 \times 10^{-4} A) 10k\Omega = +1.28V$

$= I_{at\ 2}$

③ $V_{at\ 3} = V_{at\ 2} = +1.28V$

$I_{through\ R_L} = \frac{1.28V}{1.92k\Omega} = 6.7 \times 10^{-4} A$

④ $I_{at\ 3}$ must come from $I_{at\ 4}$
 $\therefore V_{across\ R3} = (6.7 \times 10^{-4} A)(1k\Omega) = 0.67V$
 $\therefore V_{at\ 4} = 0.67V + 1.28V = 1.95V$

Summary: 3 pts. each

V	I
① 0	$1.28 \times 10^{-4} A$
② $+1.28V$	$1.28 \times 10^{-4} A$
③ $+1.28V$	$6.7 \times 10^{-4} A$
④ $+1.95V$	$6.7 \times 10^{-4} A + 1.95V$