Name: Solutions

University of California, Berkeley
Department of Materials Science & Engineering
(Total 40 pts.)
Time: 40 minutes

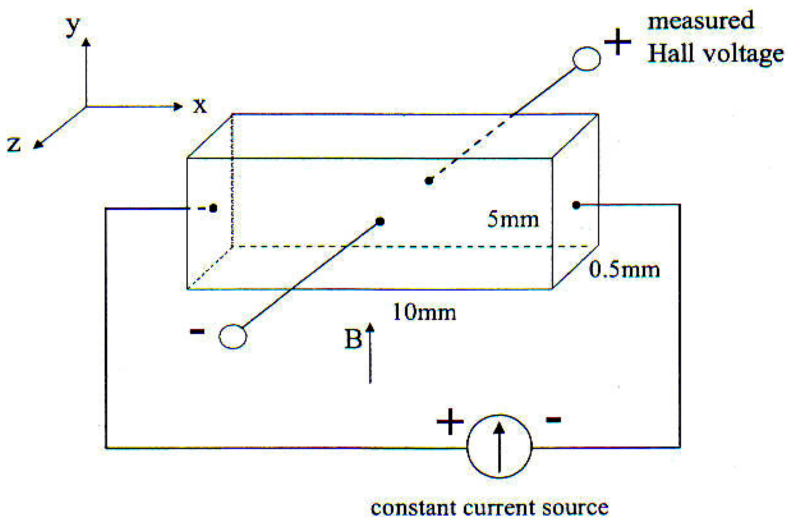
MSE 111
Prof. E. E. Haller
Quiz #1

Spring 2007

February 20, 2007

1. Hall Effect (10 points)

You make a Hall effect measurement of a GaAs sample ($10 \times 5 \times 0.5\ mm^3$) with magnetic field at right angles to the bias voltage and the measured Hall voltage, as shown below:



When you apply a magnetic field B=5.0 T in the y-direction (1 Tesla = $1 Vs/m^2$) and a current I=100mA in the x-direction, you measure the Hall voltage in the z-direction to be 10mV with the back surface being positive.

a) Determine the free carrier type and concentration.

n-type $\quad J = \frac{I}{hw} = \frac{0.1A}{(0.005m)(0.0005m)} = 4\times10^4\ A/m^2, \quad E_H = \frac{V_H}{w} = \frac{0.01V}{0.0005m} = 20V/m$

$n = \frac{JB}{eE_H} = \frac{(4\times10^4\ A/m^2)(5Vs/m^2)}{(1.6\times10^{-19}\ C)(20V/m)} = 6.25\times10^{22}\ m^{-3} = \boxed{6.25\times10^{16}\ cm^{-3}}$

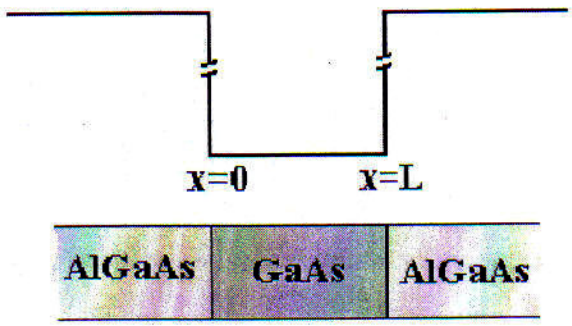
b) You now use a voltmeter (not shown) to measure a voltage of 2.0V across the sample in the x direction. Determine the resistivity and mobility of the GaAs sample.

$R = \frac{V}{I} = \frac{2V}{0.1A} = 20\ \Omega; \quad \rho = R\frac{A}{L} = 20\Omega \frac{0.005\times0.0005\ m^2}{0.01m} = 0.005\ \Omega m = \boxed{0.5\ \Omega cm}$

$\mu = \frac{1}{ne\rho} = 1/(6.25\times10^{22}\ m^{-3})(1.6\times10^{-19}C)(0.005\ \Omega m) = 0.02\ \frac{m^2}{Vs} = \boxed{200\ \frac{cm^2}{Vs}}$

2. Schrödinger Equation – Bound States (10 points)

The emission in a "Quantum Cascade Laser" is due to electronic transitions from the n=2 state to the ground state in a quantum well composed of a GaAs layer of thickness L sandwiched between two layers of AlGaAs, as shown below:



Assume that the potential is zero in the GaAs layer ($V = 0$ for $0<x<L$) and is infinite in the neighboring AlGaAs layers ($V=\infty$ for $x \leq 0$ and $x \geq L$).
This particular laser emits photons of $\lambda = 248$ nm. Calculate the thickness of the GaAs layer. (Use the electron effective mass $m_e^* = 0.066 m_e$ for GaAs.)

$$E_2 - E_1 = \frac{hc}{\lambda}$$

$$E_n = \frac{n^2 h^2}{8 m^* L^2} \Rightarrow \frac{3h^2}{8 m^* L^2} = \frac{hc}{\lambda}$$

$$\Rightarrow L = \left(\frac{3\lambda h}{8 m^* c}\right)^{1/2}$$

$$= \left(\frac{3 \times 248 \times 10^{-9} \times 6.63 \times 10^{-34}}{8 \times 0.066 \times 9.11 \times 10^{-31} \times 3 \times 10^{8}}\right)^{1/2}$$

$$\Rightarrow \boxed{L = 1.85 \text{ nm}}$$

3. Heisenberg Uncertainty Principle (10 points)
The dimensions of active regions in transistors used in our computer microprocessors have been shrinking rapidly over the past few decades. However, it has been suggested that there are fundamental limits to this scaling process. From statistical thermodynamics, it is known that the minimum energy required to switch states in any two-state system is:

$$E_{switch} = k_B T \times \ln(2)$$

Based on this switching energy, what is the smallest spatial dimension (Δx) such a two-state system can occupy? (Consider room temperature operation and use the free electron mass)

$$\Delta x . \Delta p = \frac{\hbar}{2}$$

$$E_{switch} = \frac{p^2}{2m}$$

$$\Rightarrow p = \sqrt{2m E_{switch}} = \sqrt{2m k_B T \ln 2}$$

$$\Rightarrow \Delta x = \frac{\hbar}{2\sqrt{2m k_B T \ln 2}}$$

$$\left(\hbar = \frac{h}{2\pi}\right)$$

$$= \frac{6.626\times10^{-34}}{2\pi\times2\times\sqrt{2\times9.11\times10^{-31}\times1.38\times10^{-23}\times300\times\ln 2}}$$

$$\Rightarrow \boxed{\Delta x = 0.73 \text{ nm}}$$

4. Short Answer (10 points)

- Describe bonding in graphite: covalent bonding within graphene sheets
  ______________________ van der Waals bonding between graphene sheets

- True or False: Optical phonons can exist in silicon True

- The potential between two bonded atoms can in general be expressed as $\phi(r) = \frac{A}{r^m} - \frac{B}{r^n}$. Which is a larger positive integer, m or n? m

- What is the probability to find an electron at the Fermi energy? 1/2

- True or False: The resistivity in metals increases with increasing temperature True