Problem 3

(a)

Considering that $r_2 - r_1 = d\sin\theta$, and $r_2 + r_1 = 2\bar{r}$, we get:

$$\begin{aligned}
E_{tot}(\bar{r},\theta,t) &= E_1(r_1,t) + E_2(r_1,t) && (1)\\
&= A(\bar{r})\left[\cos(\omega t + \phi_1 - kr_1) + \cos(\omega t + \phi_2 - kr_2)\right] && (2)\\
&= 2A(\bar{r})\cos\left(\omega t + \frac{\phi_1+\phi_2}{2} - k\frac{r_1+r_2}{2}\right)\cos\left(\frac{\phi_1-\phi_2}{2} - k\frac{r_1-r_2}{2}\right) && (3)\\
&= 2A(\bar{r})\cos\left(\omega t + \frac{\phi_1+\phi_2}{2} - k\bar{r}\right)\cos\left(\frac{\phi_1-\phi_2}{2} + k\frac{d\sin\theta}{2}\right) && (4)\\
& && (5)
\end{aligned}$$

(b)

$$\begin{aligned}
\left[E_{tot}(\bar{r},\theta,t)\right]^2 &= \left[2A(\bar{r})\cos\left(\omega t + \frac{\phi_1+\phi_2}{2} - k\bar{r}\right)\cos\left(\frac{\phi_1-\phi_2}{2} + k\frac{d\sin\theta}{2}\right)\right]^2 && (6)\\
&= 4A(\bar{r})^2\cos^2\left(\frac{\phi_1-\phi_2}{2} + k\frac{d\sin\theta}{2}\right)\cos^2\left(\omega t + \frac{\phi_1+\phi_2}{2} - k\bar{r}\right) && (7)\\
&= A(\bar{r})^2\cdot\frac{1+\cos\left[\phi_1-\phi_2+k(d\sin\theta)\right]}{2}\cdot\frac{1+\cos\left(2\omega t+\phi_1+\phi_2-2k\bar{r}\right)}{2} && (8)
\end{aligned}$$

Only the last term in fraction form varies with respect to time. From the properties of the Cosine function, we know that $\langle\frac{1+\cos(2\omega t+\phi_1+\phi_2-2k\bar{r})}{2}\rangle = \frac{1}{2}$. Therefore,

$$\langle\left[E_{tot}(\bar{r},\theta)\right]^2\rangle = A(\bar{r})^2\left\{1+\cos\left[\phi_1-\phi_2+k(d\sin\theta)\right]\right\}. \qquad (9)$$

(c)

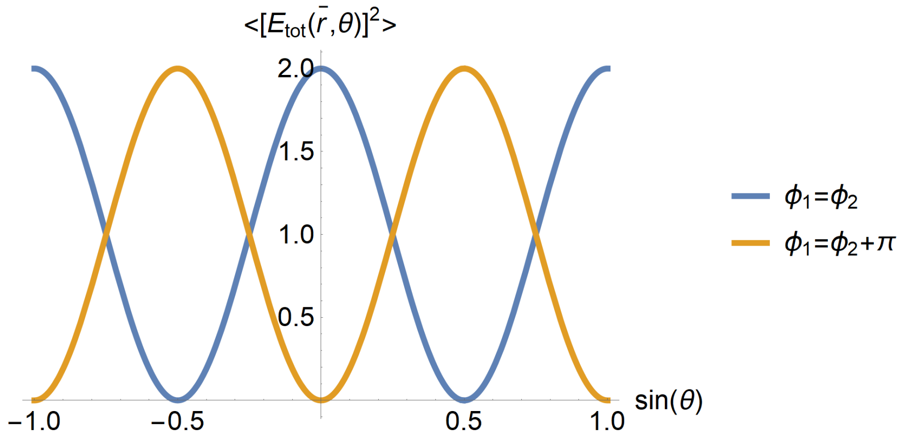

Figure 1: The horizontal axis is in units of $\frac{2\pi}{kd} = \frac{\lambda}{d}$. The vertical axis is in units of $A(\bar{r})^2$.

(d)

Let's look back at $\left[E_{tot}(\bar{r},\theta,t)\right]^2$ in part (b):

$$\left[E_{tot}(\bar{r},\theta,t)\right]^2 = A(\bar{r})^2 \cdot \frac{1+\cos\left[\phi_1-\phi_2+k(d\sin\theta)\right]}{2} \cdot \frac{1+\cos\left(2\omega t+\phi_1+\phi_2-2k\bar{r}\right)}{2} \tag{10}$$

When $\phi_1-\phi_2$ fluctuates with respect to time, both of the fractions in the above expression will give a long-time average of 1/2. Therefore, the long-time averaged value will become:

$$\langle\left[E_{tot}(\bar{r},\theta)\right]^2\rangle = A(\bar{r})^2. \tag{11}$$

All the θ dependencies are erased clean.

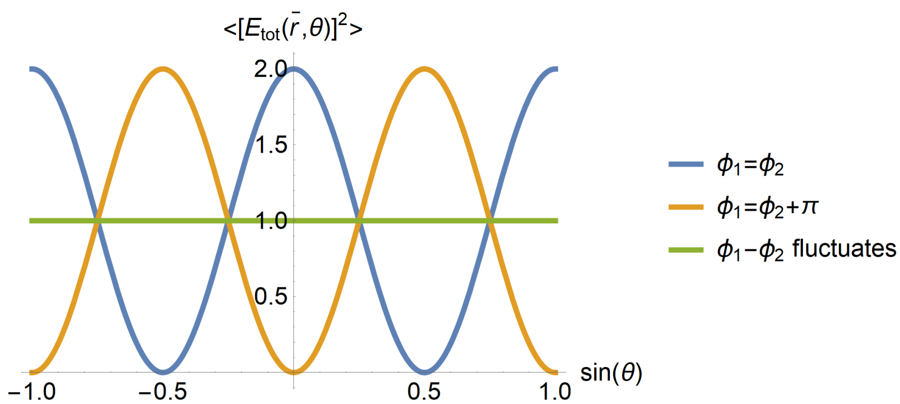

Figure 2: The horizontal axis is in units of $\frac{2\pi}{kd} = \frac{\lambda}{d}$. The vertical axis is in units of $A(\bar{r})^2$.

Problem 2 (30 points)

a)

In this case, the distance to the object is $d_o = d_f$ and, since the image is formed at the retina, the distance to the image is $d_i = L_0$. Now, we use the lens equation

$$\frac{1}{d_f} + \frac{1}{L_0} = \frac{1}{f_e} \rightarrow \boxed{f_e = \frac{L_0}{1 + L_0/(d_f)}.} \tag{1}$$

b)

As is said in the problem, the image of an object at infinity in the corrective lens is located a distance d_f before the lens. Thus,

$$\frac{1}{\infty} + \frac{1}{-d_f} = \frac{1}{f_g} \rightarrow \boxed{f_g = -d_f.} \tag{2}$$

c)

First, we find the location x of the image of the object created by the corrective lens

$$\frac{1}{2d_f} + \frac{1}{x} = -\frac{1}{d_f} \rightarrow x = -\frac{2d_f}{3}. \tag{3}$$

The image is located a distance $\frac{2d_f}{3}$ before the lens. Finally, we find the location y of image formed by the second lens

$$\frac{3}{2d_f} + \frac{1}{y} = \frac{d_f + L_0}{d_f L_0} \rightarrow \boxed{y = \frac{L_0}{1 - L_0/(2d_f)}}. \tag{4}$$

The image is formed before the retina.

d)

The eyeball's actual focal length adjusts to create the image at the retina:

$$\frac{3}{2d_f} + \frac{1}{L_0} = \frac{1}{f_1} \rightarrow \boxed{f_1 = \frac{L_0}{1 + 3L_0/(2d_f)}}. \tag{5}$$

It is less than f_e.

Midterm 1 Solutions - Problems 1 and 4

February 28, 2020

Problem 1

a)

We're given that the total power emitted from the rod is $\dot{E}$, and since the shell absorbs all radiation, the total radiation in the volume in question is just that from the rod. We also know that $\dot{E} = IA$, where $I = \frac{1}{2}c\epsilon_0 E^2$ is the intensity, or time averaged energy flux density, of the electromagnetic radiation. Ignoring the top and bottom areas of the rod, we have

$$\dot{E} = \frac{1}{2}c\epsilon_0 E^2 (2\pi r H)$$

so at $r = r_1$,

$$E_1 = \sqrt{\frac{\dot{E}}{\pi r_1 H c\epsilon_0}} \tag{1}$$

b)

This is the same as in part a), but $r_1 \to r_2$, so

$$E_2 = \sqrt{\frac{\dot{E}}{\pi r_2 H c\epsilon_0}} \tag{2}$$

$$= E_1\sqrt{\frac{r_1}{r_2}} \tag{3}$$

c)

The energy density of electromagnetic fields is given by

$$u = \frac{1}{2}\epsilon_0 E^2$$

and so from equation (2) we have

$$u_2 = \frac{\dot{E}}{2\pi r_2 H c} \tag{4}$$

d)

The radiation pressure exerted on a perfect absorber is

$$P_{rad} = \frac{I}{c}$$

and since the wall is at r_2, we have

$$P_{rad}(r_2) = \frac{I_2}{c} = \frac{\frac{1}{2}c\epsilon_0 E_2^2}{c} = \frac{\dot{E}}{2\pi r_2 H c} = u_2 \tag{5}$$

So in this case, the radiation pressure at r_2 is the same as the energy density.

Problem 4

a)

First, the distance between the slits is given by

$$d = \frac{W}{N}$$

We are asked to find the distance between bright spots of an interference pattern, S_0. Thus our equation becomes

$$d \sin\theta = m\lambda_0 \tag{6}$$

Assuming small angle approximation, we have

$$\theta \simeq \frac{\lambda_0}{d} = \frac{\lambda_0 N}{W} \tag{7}$$

for m = 1. We can relate S_0 to θ by noting that S_0 lies on the opposite side of the right triangle with D as the adjacent side. Therefore

$$\tan\theta = S_0/D$$

and so, using the small angle approximation again,

$$S_0 = D\tan\theta \simeq D\theta = \frac{D\lambda_0 N}{W} \tag{8}$$

b)

Increasing N increases the number of interference minima between principal maxima (the stripes), so ΔS_0 becomes smaller. The angle to the first minimum is

$$Nd\sin\theta = \lambda_0 = W\sin\theta \implies \theta = \sin^{-1}\frac{\lambda_0}{W} \simeq \frac{\lambda_0}{W}$$

After noting that ΔS_0 is twice the distance from the center to the first minimum, we get

$$\tan\theta = \frac{\Delta S_0}{2D}$$

and so

$$\Delta S_0 = 2D\tan\theta \simeq 2D\frac{\lambda_0}{W} \tag{9}$$

c)

If Δy is the separation between the first stripes above the central maximum of each wavelength, then the minimum value of $\Delta\lambda$ that will allow us to distinguish them apart will be such that Δy is equal to half of the width of the strip, ΔS_0 (in other words, the separation of the peaks must be at least as far as the distance between the peak and the first minimum to either side of it). Luckily for us, we've already calculated the stripe width (eq. 8) and the distance between stripes (7). Defining

$$S_\Delta = D(\lambda_0 + \Delta\lambda)N/W$$

as the distance between stripes for the $\lambda_0 + \Delta\lambda$ light, we have

$$S_\Delta - S_0 = \frac{D\Delta\lambda N}{W} + \frac{D\lambda_0 N}{W} - \frac{D\lambda_0 N}{W} \tag{10}$$

$$= \frac{D\Delta\lambda N}{W} = \Delta y = \frac{D\lambda_0}{W} \tag{11}$$

$$\implies \Delta\lambda = \frac{\lambda_0}{N} \tag{12}$$

d)

It's the same as in part (c), but now our distances from the center are $2S_0$ and $2S_\Delta$, so

$$2S_\Delta - 2S_0 = \frac{2D\Delta\lambda N}{W} = \Delta y = \frac{D\lambda_0}{W} \tag{13}$$

$$\implies \Delta\lambda = \frac{\lambda_0}{2N} \tag{14}$$