

Chemistry 4B, Exam II

Name \_\_\_\_\_

March 9, 2020

SID \_\_\_\_\_

Professor R. J. Saykally

TA \_\_\_\_\_

$$E = mc^2$$

$$1 \text{ day} = 86400 \text{ sec}$$

$$N_t = N_0 \exp(-kt)$$

$$w_{\text{elec,rev}} = \Delta G = -nF\Delta E_{\text{cell}}$$

$$A = kN$$

$$\Delta E_{\text{cell}}^0 = E_{\text{cathode}}^0 - E_{\text{anode}}^0$$

$$1 \text{ Gy} = 1 \text{ J kg}^{-1}$$

$$\Delta E_{\text{cell}} = \Delta E_{\text{cell}}^0 - \frac{0.0592 \text{ V}}{n} \log_{10} Q$$

1 Bq  $\equiv$  1 disintegration per second

**Rules:**

1. **Show *all work* for full credit and partial credit.** This includes:  
Relevant equation(s), intermediate steps, substitution of values *with units*, unit conversions
2. Work all problems to **correct # of sig figs**
3. No lecture notes or books permitted
4. No word processing calculators (including graphing calculators)
5. No cell phones
6. No smart devices (e.g., phones, watches, etc.)
7. Time: 50 minutes
8. Equations, Physical Constants and Conversion Factors, Masses of Select Elementary Particles, and Standard Reduction Potentials included

1 (a) The half-lives of  $^{235}\text{U}$  and  $^{238}\text{U}$  are  $7.04 \times 10^8$  years and  $4.47 \times 10^9$  years, respectively, and the present abundance ratio is  $^{238}\text{U}/^{235}\text{U} = 137.7$ . It is thought that their abundance ratio was 1 at some time *before* our Earth and solar system were formed ( $4.5 \times 10^9$  years ago). Estimate how long ago the supernova occurred that produced all the uranium isotopes in equal abundance, including the two longest lived isotopes,  $^{238}\text{U}$  and  $^{235}\text{U}$ .

$$N(^{235}\text{U}) = N_i(^{235}\text{U})e^{-kt} \text{ and } N(^{238}\text{U}) = N_i(^{238}\text{U})e^{-kt}$$

$$\frac{137.7}{1} = \frac{N_i(^{238}\text{U})e^{-k_{238}t}}{N_i(^{235}\text{U})e^{-k_{235}t}} = \frac{e^{-k_{238}t}}{e^{-k_{235}t}}$$

$$\ln(137.7) = k_{235}t - k_{238}t = t(k_{235} - k_{238})$$

$$k = \frac{\ln(2)}{t_{1/2}}$$

$$\ln(137.7) = t \left( \frac{\ln(2)}{7.04 \times 10^8 \text{ yr}} - \frac{\ln(2)}{4.47 \times 10^9} \right)$$

$$t = \frac{\ln(137.7)}{\left( \frac{\ln(2)}{7.04 \times 10^8 \text{ yr}} - \frac{\ln(2)}{4.47 \times 10^9 \text{ yr}} \right)} = 5.93 \times 10^9 \text{ yr}$$

(b) Using the above result and the accepted age of the Earth ( $4.5 \times 10^9$  yr), calculate the  $^{238}\text{U}/^{235}\text{U}$  ratio at the time Earth was formed.

$$\frac{N(^{235}\text{U})}{N_i(^{235}\text{U})} = e^{-k_{235}t} \text{ and } \frac{N(^{238}\text{U})}{N_i(^{238}\text{U})} = e^{-k_{238}t}$$

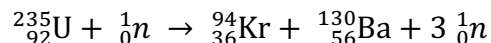
$$\frac{N(^{238}\text{U})}{N(^{235}\text{U})} = \frac{e^{-k_{238}t}}{e^{-k_{235}t}} = e^{t(k_{235} - k_{238})}$$

$$t = 5.93 \times 10^9 \text{ yr} - 4.5 \times 10^9 \text{ yr} = 1.43 \times 10^9 \text{ yr}$$

$$\ln \left( \frac{N(^{238}\text{U})}{N(^{235}\text{U})} \right) = (1.43 \times 10^9 \text{ yr}) * \left( \frac{\ln(2)}{7.04 \times 10^8 \text{ yrs}} - \frac{\ln(2)}{4.47 \times 10^9 \text{ yrs}} \right)$$

$$\frac{N(^{238}\text{U})}{N(^{235}\text{U})} = 3.3$$

2 (a) Calculate the amount of energy released, in kilojoules per *gram* of Uranium, in the fission reaction:



Use the atomic masses in Table 19.1. The atomic mass of  ${}^{94}\text{Kr}$  is 93.919 u, that of  ${}^{139}\text{Ba}$  is 138.909 u, and that of  ${}^{235}\text{U}$  is 235.044 u.

$$\Delta E = \Delta mc^2$$

$$\Delta m = m({}^{94}\text{Kr}) + m({}^{139}\text{Ba}) + 3 * m({}^1_0n) - m({}^{235}_{92}\text{U}) - m({}^1_0n)$$

$$\Delta m = 93.919 \text{ u} + 138.909 \text{ u} + 3 * (1.008665) \text{ u} - 235.044 \text{ u} - 1.008665 \text{ u} = -0.19867 \text{ u}$$

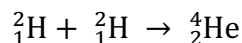
$$\Delta E = -0.19867 \text{ u} * (2.998 \times 10^8 \text{ m s}^{-1})^2 * \frac{1.6605 \times 10^{-27} \text{ kg}}{\text{u}} = -2.964 \times 10^{-11} \text{ J}$$

$$\Delta E = -2.964 \times 10^{-11} \text{ J} * \left( \frac{1 \text{ kJ}}{1000 \text{ J}} \right) * 6.023 \times 10^{23} \text{ mol}^{-1} = -1.784 \times 10^{10} \text{ kJ mol}^{-1}$$

$$\Delta E = -1.784 \times 10^{10} \text{ kJ mol}^{-1} * \left( \frac{1 \text{ mol } {}^{235}\text{U}}{235.044 \text{ g } {}^{235}\text{U}} \right) = -7.592 \times 10^7 \text{ kJ g}^{-1}$$

$$\text{Energy released} = +7.592 \times 10^7 \text{ kJ g}^{-1}$$

(b) Using Table 19.1. Calculate the amount of energy released, in kilojoules per *gram* of deuterium ( ${}^2\text{H}$ ), for the fusion reaction:



$$\Delta m = m({}^4_2\text{He}) - 2 * m({}^2_1\text{H}) = 4.00260325 - 2(2.014101778) = -0.0256003 \text{ u}$$

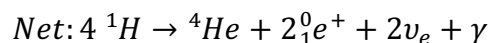
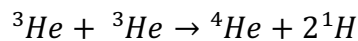
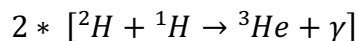
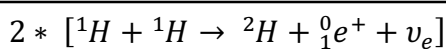
$$\Delta E = -0.0256003 \text{ u} * (2.998 \times 10^8 \text{ m s}^{-1})^2 * \frac{1.6605 \times 10^{-27} \text{ kg}}{\text{u}} = -3.28074 \times 10^{-12} \text{ J}$$

$$\Delta E = -3.28074 \times 10^{-12} \text{ J} * \left( \frac{1 \text{ kJ}}{1000 \text{ J}} \right) * 6.023 \times 10^{23} \frac{\text{atoms}}{\text{mol}} * \frac{1}{2 \text{ atoms}} = -1.15004 \times 10^9 \text{ kJ mol}^{-1}$$

$$\Delta E = -1.15004 \times 10^9 \text{ kJ mol}^{-1} * \left( \frac{1 \text{ mol } {}^2_1\text{H}}{2.01410 \text{ g } {}^2_1\text{H}} \right) = -5.712 \times 10^8 \text{ kJ g}^{-1}$$

$$\text{Energy released} = +5.712 \times 10^8 \text{ kJ g}^{-1}$$

3) (a) Write the equations describing the p-p fusion cycle that provides 90% of the sun's energy, with appropriate multipliers to give the correct overall fusion reaction.



(b) Compute  $\Delta m$  and  $\Delta E$  for the overall "hydrogen burning" fusion cycle.

$$\Delta m = m({}^4_2\text{He}) + 2 * m({}^0_1\text{e}^+) - 4 * m({}^1_1\text{H}) = 4.002603 + 2(0.00054857) - 4(1.007825)$$

$$\Delta m = -0.02759 \text{ u}$$

$$\Delta E = -0.02759 \text{ u} * (2.998 \times 10^8 \text{ m s}^{-1})^2 * \frac{1.6605 \times 10^{-27} \text{ kg}}{\text{u}} = -4.119 \times 10^{-12} \text{ J}$$

$$\Delta E = -4.119 \times 10^{-12} \text{ J} * \left(\frac{1 \text{ kJ}}{1000 \text{ J}}\right) * 6.023 \times 10^{23} \frac{\text{atoms}}{\text{mol}^{-1}} * \frac{1}{4 \text{ atoms}} = -6.1993 \times 10^8 \text{ kJ mol}^{-1}$$

$$\Delta E = -6.1993 \times 10^8 \text{ kJ mol}^{-1} * \left(\frac{1 \text{ mol } {}^1_1\text{H}}{1.007825 \text{ g } {}^1_1\text{H}}\right) = -6.151 \times 10^8 \text{ kJ g}^{-1} \text{H}$$

$$\Delta E = -6.151 \times 10^8 \text{ kJ g}^{-1}$$

(c) Our sun radiates  $3.9 \times 10^{23}$  Watts of power. How many protons must be "burned" each second to supply this energy

$$\frac{3.9 \times 10^{23} \frac{\text{J}}{\text{s}}}{4 \text{ atoms} \cdot 6.151 \times 10^8 \frac{\text{kJ}}{\text{g}}} = 3.8 \times 10^{35} \frac{\text{protons}}{\text{second}}$$

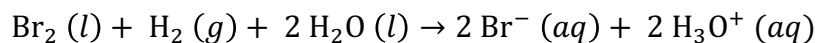
or

$$3.9 \times 10^{23} \text{ W} = 3.9 \times 10^{23} \frac{\text{J}}{\text{s}} = 3.9 \times 10^{20} \frac{\text{kJ}}{\text{s}}$$

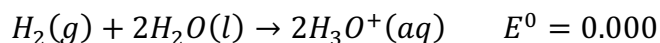
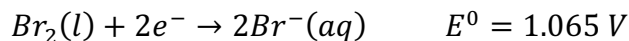
$$\frac{3.9 \times 10^{20} \frac{\text{kJ}}{\text{s}}}{6.151 \times 10^8 \frac{\text{kJ}}{\text{g}}} = 6.3404 \times 10^{11} \frac{\text{g } {}^1_1\text{H}}{\text{s}}$$

$$6.3404 \times 10^{11} \frac{\text{g } {}^1_1\text{H}}{\text{s}} * \frac{1 \text{ mol } {}^1_1\text{H}}{1.007825 \text{ g}} * \frac{6.023 \times 10^{23} \text{ atoms}}{\text{mol}} = 3.8 \times 10^{35} \frac{\text{protons}}{\text{second}}$$

4) A galvanic cell is constructed in which the overall reaction is



(a) Calculate  $\Delta E^\circ$  for this cell.



$$\Delta E_{\text{cell}}^\circ = E_{\text{cathode}}^\circ - E_{\text{anode}}^\circ = 1.065 - 0.00 = 1.065 \text{ V}$$

(b) Silver ions are added until AgBr precipitates at the cathode and  $[\text{Ag}^+]$  reaches 0.060 M. The cell potential is then measured to be 1.710 V at pH = 0 and  $P_{\text{H}_2} = 1.0$  atm. Calculate  $[\text{Br}^-]$  under these conditions.

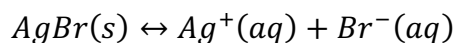
$$\Delta E_{\text{cell}} = \Delta E_{\text{cell}}^\circ - \frac{0.0592 \text{ V}}{n} \log_{10} Q$$

$$1.710 \text{ V} = 1.065 \text{ V} - \frac{0.0592 \text{ V}}{2} \log_{10} \frac{[\text{Br}^-]^2 * [\text{H}_3\text{O}^+]^2}{P_{\text{H}_2}}$$

$$-21.7905 = \log_{10} [\text{Br}^-]^2$$

$$[\text{Br}^-] = \sqrt{10^{-21.7905}} = 1.3 \times 10^{-11} \text{ M}$$

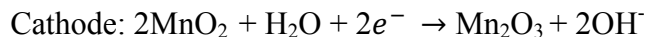
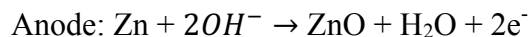
(c) Calculate the solubility product constant  $K_{\text{sp}}$  for AgBr.



$$K_{\text{sp}} = [\text{Ag}^+][\text{Br}^-]$$

$$K_{\text{sp}} = [0.060][1.27 \times 10^{-11}] = 7.6 \times 10^{-13}$$

5) A modern size AA alkaline dry cell battery weighs 23.0 grams and produces 1.50 volts with the following half-reactions:



- (a) Assuming that 70.0% of its mass consists of the cell reagents in stoichiometric amounts, how much charge can this battery deliver?

$$23.0 \text{ g} * 0.700 = 16.1 \text{ g}$$

$$M(\text{cell}) = M(\text{Zn}) + 2M(\text{MnO}_2) = 65.38 + 2(86.936) = 239.25 \frac{\text{g}}{\text{mol}}$$

$$\frac{16.1 \text{ g}}{239.25 \frac{\text{g}}{\text{mol}}} = 0.06729 \text{ mol Zn} * \frac{2 \text{ mol e}^-}{1 \text{ mol Zn}} = 0.1345 \text{ mol e}^-$$

$$Q = nF = (0.1345 \text{ mol e}^-) \left( 96485 \frac{\text{C}}{\text{mol e}^-} \right) = 1.30 \times 10^4 \text{ C}$$

- (b) How much work can it do?

$$w = -nF\Delta E_{\text{cell}}$$

$$w = (-1.30 \times 10^4 \text{ C}) * (1.50 \text{ V}) = -1.95 \times 10^4 \text{ J} = -19.5 \text{ kJ}$$

- (c) If a steady current of 0.050 A is drawn from it, how long will it last?

$$0.050 \text{ A} = 0.050 \frac{\text{C}}{\text{s}}$$

$$\frac{1.30 \times 10^4 \text{ C}}{0.050 \frac{\text{C}}{\text{s}}} = 2.6 \times 10^5 \text{ s}$$