Chemistry 4B, Exam II Name ________________________________

March 9, 2020 SID ________________________________

Professor R. J. Saykally TA ________________________________

$$E = mc^2$$

$$1 \text{ day} = 86400 \text{ sec}$$

$$N_t = N_0 \exp(-kt)$$

$$w_{\text{elec,rev}} = \Delta G = -nF\Delta E_{\text{cell}}$$

$$A = kN$$

$$\Delta E^0_{\text{cell}} = E^0_{\text{cathode}} - E^0_{\text{anode}}$$

$$1 \text{ Gy} = 1 \text{ J kg}^{-1}$$

$$\Delta E_{\text{cell}} = \Delta E^0_{\text{cell}} - \frac{0.0592 \text{ V}}{n} \log_{10} Q$$

1 Bq ≡ 1 disintegration per second

<u>Rules</u>:

1. **Show *all work* for full credit and partial credit.** This includes: Relevant equation(s), intermediate steps, substitution of values *with units*, unit conversions
2. Work all problems to ***correct # of sig figs***
3. No lecture notes or books permitted
4. No word processing calculators (including graphing calculators)
5. No cell phones
6. No smart devices (e.g., phones, watches, etc.)
7. Time: 50 minutes
8. Equations, Physical Constants and Conversion Factors, Masses of Select Elementary Particles, and Standard Reduction Potentials included

1 (a) The half-lives of ^{235}U and ^{238}U are 7.04 x 10^8 years and 4.47 x 10^9 years, respectively, and the present abundance ratio is $^{238}U/^{235}U$ = 137.7. It is thought that their abundance ratio was 1 at some time ***before*** our Earth and solar system were formed (4.5 x 10^9 years ago). Estimate how long ago the supernova occurred that produced all the uranium isotopes in equal abundance, including the two longest lived isotopes, ^{238}U and ^{235}U.

$$N(^{235}U) = N_i(^{235}U)e^{-kt} \text{ and } N(^{238}U) = N_i(^{238}U)e^{-kt}$$

$$\frac{137.7}{1} = \frac{N_i(^{238}U)e^{-k_{238}t}}{N_i(^{235}U)e^{-k_{235}t}} = \frac{e^{-k_{238}t}}{e^{-k_{235}t}}$$

$$\ln(137.7) = k_{235}t - k_{238}t = t(k_{235} - k_{238})$$

$$k = \frac{\ln(2)}{t_{1/2}}$$

$$\ln(137.7) = t\left(\frac{\ln(2)}{7.04x10^8 yr} - \frac{\ln(2)}{4.47x10^9}\right)$$

$$t = \frac{\ln(137.7)}{\left(\frac{\ln(2)}{7.04x10^8 yr} - \frac{\ln(2)}{4.47x10^9 yr}\right)} = 5.93x10^9 yr$$

(b) Using the above result and the accepted age of the Earth (4.5 x 10^9 yr), calculate the $^{238}U/^{235}U$ ratio at the time Earth was formed.

$$\frac{N(^{235}U)}{N_i(^{235}U)} = e^{-k_{235}t} \text{ and } \frac{N(^{238}U)}{N_i(^{238}U)} = e^{-k_{238}t}$$

$$\frac{N(^{238}U)}{N(^{235}U)} = \frac{e^{-k_{238}t}}{e^{-k_{235}t}} = e^{t(k_{235}-k_{238})}$$

$$t = 5.93x10^9 yr - 4.5x10^9 yr = 1.43x10^9 yr$$

$$\ln\left(\frac{N(^{238}U)}{N(^{235}U)}\right) = (1.43x10^9 yr) * \left(\frac{\ln(2)}{7.04x10^8 yrs} - \frac{\ln(2)}{4.47x10^9 yrs}\right)$$

$$\frac{N(^{238}U)}{N(^{235}U)} = 3.3$$

2 (a) Calculate the amount of energy released, in kilojoules per *gram* of Uranium, in the fission reaction:

$$^{235}_{92}\text{U} + {}^{1}_{0}n \rightarrow {}^{94}_{36}\text{Kr} + {}^{130}_{56}\text{Ba} + 3\,{}^{1}_{0}n$$

Use the atomic masses in Table 19.1. The atomic mass of ^{94}Kr is 93.919 u, that of ^{139}Ba is 138.909 u, and that of ^{235}U is 235.044 u.

$$\Delta E = \Delta mc^2$$

$$\Delta m = m({}^{94}_{36}Kr) + m({}^{130}_{56}Ba) + 3 * m({}^{1}_{0}n) - m({}^{235}_{92}92) - m({}^{1}_{0}n)$$

$$\Delta m = 93.919\,u + 138.909\,u + 3 * (1.008665)u - 235.044\,u - 1.008665\,u = -0.19867\,u$$

$$\Delta E = -0.19867\,u * (2.998x10^{8} m\,s^{-1})^2 * \frac{1.6605x10^{-27} kg}{u} = -2.964x10^{-11} J$$

$$\Delta E = -2.964x10^{-11} J * \left(\frac{1kJ}{1000\,J}\right) * 6.023x10^{23} mol^{-1} = -1.784x10^{10} kJ\,mol^{-1}$$

$$\Delta E = -1.784x10^{10} kJ\,mol^{-1} * \left(\frac{1\,mol\,{}^{235}U}{235.044\,g\,{}^{235}U}\right) = -7.592x10^{7}\,kJ\,g^{-1}$$

Energy released = +7.592x10^{7} kJ g^{-1}

(b) Using Table 19.1. Calculate the amount of energy released, in kilojoules per *gram* of deuterium (^{2}H), for the fusion reaction:

$$^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He}$$

$$\Delta m = m({}^{4}_{2}\text{He}) - 2 * m({}^{2}_{1}H) = 4.00260325 - 2(2.014101778) = -0.0256003\,u$$

$$\Delta E = -0.0256003\,u * (2.998x10^{8} m\,s^{-1})^2 * \frac{1.6605x10^{-27} kg}{u} = -3.28074x10^{-12} J$$

$$\Delta E = -3.28074x10^{-12} J * \left(\frac{1kJ}{1000\,J}\right) * 6.023x10^{23} \frac{atoms}{mol^{-1}} * \frac{1}{2\,atoms} = -1.15004x10^{9} kJ\,mol^{-1}$$

$$\Delta E = -1.15004x10^{9} kJ\,mol^{-1} * \left(\frac{1\,mol\,{}^{2}_{1}H}{2.01410\,g\,{}^{2}_{1}H}\right) = -5.712x10^{8}\,kJ\,g^{-1}$$

Energy released = + 5.712 x 10^{8} kJ g^{-1}

3) (a) Write the equations describing the p-p fusion cycle that provides 90% of the sun's energy, with appropriate multipliers to give the correct overall fusion reaction.

$$2*\ [{}^{1}H + {}^{1}H \rightarrow {}^{2}H + {}^{0}_{1}e^{+} + \upsilon_e]$$

$$2*\ [{}^{2}H + {}^{1}H \rightarrow {}^{3}He + \gamma]$$

$$\underline{{}^{3}He + {}^{3}He \rightarrow {}^{4}He + 2\,{}^{1}H}$$

$$Net\!: 4\ {}^{1}H \rightarrow {}^{4}He + 2\,{}^{0}_{1}e^{+} + 2\upsilon_e + \gamma$$

(b) Compute Δm and ΔE for the overall "hydrogen burning" fusion cycle.

$$\Delta m = m({}^{4}He) + 2*m({}^{0}_{1}e^{+}) - 4*m({}^{1}H) = 4.002603 + 2(0.00054857) - 4(1.007825)$$

$$\Delta m = -0.02759\ u$$

$$\Delta E = -0.02759\ u*(2.998x10^{8} m\ s^{-1})^{2} * \frac{1.6605x10^{-27} kg}{u} = -4.119x10^{-12} J$$

$$\Delta E = -4.119x10^{-12} J * \left(\frac{1 kJ}{1000\ J}\right) * 6.023x10^{23} \frac{atoms}{mol^{-1}} * \frac{1}{4\ atoms} = -6.1993x10^{8} kJ\ mol^{-1}$$

$$\Delta E = -6.1993x10^{8} kJ\ mol^{-1} * \left(\frac{1\ mol\ {}^{1}_{\square}H}{1.007825\ g\ {}^{1}_{1}H}\right) = -6.151x10^{8}\ kJ\ g^{-1}\,{}^{1}H$$

$$\Delta E = -6.151x10^{8} kJ\ g^{-1}$$

(c) Our sun radiates 3.9 x 10^{23} Watts of power. How many protons must be "burned" each second to supply this energy

$$\frac{3.9x10^{23}\frac{J}{s}}{\frac{4.119x10^{-12} J}{4\ atoms}} = 3.8x10^{35} \frac{protons}{second}$$

or

$$3.9x10^{23}\ W = 3.9x10^{23} \frac{J}{s} = 3.9x10^{20} \frac{kJ}{s}$$

$$\frac{3.9x10^{20}\frac{kJ}{s}}{6.151x10^{8}\frac{kJ}{g}} = 6.3404x10^{11} \frac{g\ {}^{1}H}{s}$$

$$6.3404x10^{11} \frac{g\ {}^{1}H}{s} * \frac{1\ mol\,{}^{1}H}{1.0078250\ g} * \frac{6.023x10^{23} atoms}{mol}$$

$$= 3.8x10^{35} \frac{protons}{second}$$

4) A galvanic cell is constructed in which the overall reaction is

$$Br_2\ (l) + H_2\ (g) + 2\ H_2O\ (l) \rightarrow 2\ Br^-\ (aq) + 2\ H_3O^+\ (aq)$$

(a) Calculate ΔE° for this cell.

$$Br_2(l) + 2e^- \rightarrow 2Br^-(aq) \qquad E^0 = 1.065\ V$$

$$H_2(g) + 2H_2O(l) \rightarrow 2H_3O^+(aq) \qquad E^0 = 0.000$$

$$\Delta E^0_{cell} = E^0_{cathode} - E^0_{anode} = 1.065 - 0.00 = 1.065\ V$$

(b) Silver ions are added until AgBr precipitates at the cathode and $[Ag^+]$ reaches 0.060 M. The cell potential is then measured to be 1.710 V at pH = 0 and P_{H2}= 1.0 atm. Calculate $[Br^-]$ under these conditions.

$$\Delta E_{\text{cell}} = \Delta E^0_{\text{cell}} - \frac{0.0592\ \text{V}}{n} \log_{10} Q$$

$$1.710\ V = 1.065\ V - \frac{0.0592\ \text{V}}{2} \log_{10} \frac{[Br^-]^2 * [H_3O^+]^2}{P_{H_2}}$$

$$-21.7905 = log_{10}[Br^-]^2$$

$$[Br^-] = \sqrt{10^{-21.7905}} = 1.3x10^{-11}\ M$$

(c) Calculate the solubility product constant K_{sp} for AgBr.

$$AgBr(s) \leftrightarrow Ag^+(aq) + Br^-(aq)$$

$$K_{sp} = [Ag^+][Br^-]$$

$$K_{sp} = [0.060][1.27x10^{-11}] = 7.6x10^{-13}$$

5) A modern size AA alkaline dry cell battery weighs 23.0 grams and produces 1.50 volts with the following half-reactions:

Anode: $Zn + 2OH^- \rightarrow ZnO + H_2O + 2e^-$

Cathode: $2MnO_2 + H_2O + 2e^- \rightarrow Mn_2O_3 + 2OH^-$

(a) Assuming that 70.0% of its mass consists of the cell reagents in stoichiometric amounts, how much charge can this battery deliver?

$$23.0\ g * 0.700 = 16.1\ g$$

$$M(cell) = M(Zn) + +2M(MnO_2) = 65.38 + + 2(86.936) = 239.25\ \frac{g}{mol}$$

$$\frac{16.1\ g}{239.25\ \frac{g}{mol}} = 0.06729\ mol\ Zn * \frac{2\ mol\ e^-}{1\ mol\ Zn} = 0.1345\ mol\ e^-$$

$$Q = nF = (0.1345\ mol\ e^-)\left(96485\ \frac{C}{mol\ e^-}\right) = 1.30x10^4 C$$

(b) How much work can it do?

$$w = -nF\Delta E_{cell}$$

$$w = (-1.30x10^4 C) * (1.50\ V) = -1.95x10^4\ J = -19.5\ kJ$$

(c) If a steady current of 0.050 A is drawn from it, how long will it last?

$$0.050\ A = 0.050\ \frac{C}{s}$$

$$\frac{1.30x10^4 C}{0.050\ \frac{C}{s}} = 2.6x10^5 s$$