Problem 1

The trajectory of the firecracker can be described by the two 'holy grail' kinematics equations:

$$x(t) = x_0 + v_{0,x}t + \frac{1}{2}a_x t^2 \tag{1}$$

and

$$y(t) = y_0 + v_{0,y}t + \frac{1}{2}a_y t^2 \tag{2}$$

From the problem statement, we are given that $v_{0,x} = v$, $v_{0,y} = 0$, $y_0 = h$, and $a_y = -g$. We can also define $x_0 = 0$. Moreover, we know that $a_x = -a$, since a negative acceleration is required for the firecracker to turn around and return to the initial x position. Making these substitutions, we obtain that

$$x(t) = vt - \frac{1}{2}at^2 \tag{3}$$

and

$$y(t) = h - \frac{1}{2}gt^2 \tag{4}$$

Now, at the point at which the firecracker hits the ground, $x = 0$, and $y = 0$. From eq 3, we can solve for the time at which the firecracker hits the ground:

$$t = \frac{2v}{a} \tag{5}$$

and plugging into eq 4 with $y = 0$, we obtain the answer:

$$h = \frac{2v^2}{a^2}g \tag{6}$$

Problem 2

We will make use of three key pieces of information that are given in the problem statement. Working in units of $\frac{km}{hr}$ and letting v_w be the speed of the river and v_s be the speed of the students

- The students travel against stream (velocities subtract) and cover 2 km in an hour. Thus,

$$v_s - v_w = 2\ \frac{km}{hr} \tag{1}$$

- In the total time t_{tot}, the bottle floats a distance of (5-2) = 3 km, going at stream speed. Thus,

$$v_w t_{tot} = 3\ km \tag{2}$$

- and lastly we can write t_{tot} in terms of the motion students.

$$t_{tot} = 1\ hr\ + \frac{5\ km}{v_s + v_w} \tag{3}$$

  where the 1 hr is from the the first leg of their motion, and the second term is after turning around to fetch the bottle.

We can equate equations 2 and 3. Doing so (and rearranging) we get

$$v_w^2 + 2v_w = v_s(3 - v_w) \tag{4}$$

plugging in equation 1 to eliminate v_s, and rearranging, we get

$$2v_w^2 + v_w - 6 = 0 \tag{5}$$

this can be factored to give

$$(2v_w - 3)(v_w + 2) = 0 \tag{6}$$

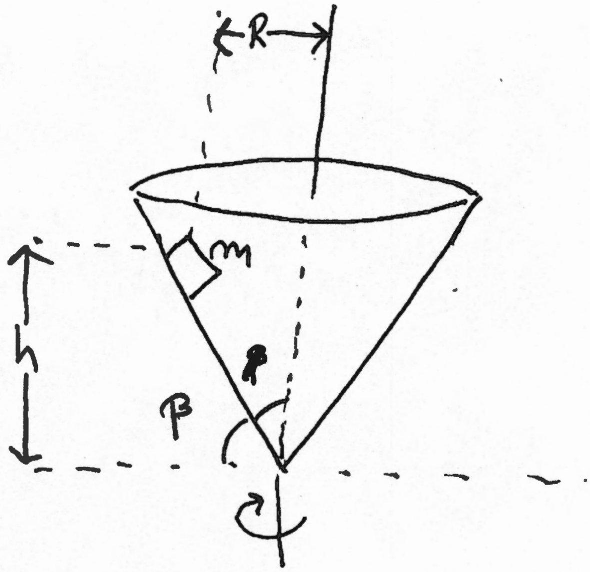
and only the first term gives a meaningful result since speeds are magnitudes, and magnitudes are positive. Thus,

$$v_w = 1.5\ \frac{km}{hr} \tag{7}$$

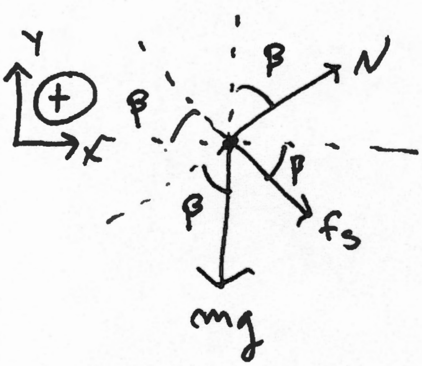
and for part b)
just plug in the above answer to equation 1 to get

$$v_s = 3.5\ \frac{km}{hr} \tag{8}$$

#3

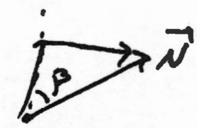


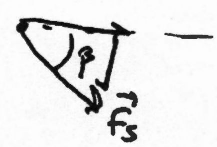
(a) For max m will want to slide up the cone.



$\vec{N} = N\sin\beta\,\hat{x} + N\cos\beta\,\hat{y}$

$\vec{f_s} = f_s\cos\beta\,\hat{x} - f_s\sin\beta\,\hat{y}$





$$T = \frac{2\pi}{\omega}$$

$$\sum F_x = N\sin\beta + f_s\cos\beta = \frac{mv^2}{R} \quad (1)$$

$V = R\omega$

$f_s = \mu_s N$

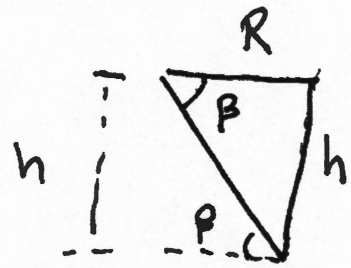
$$\sum F_y = N\cos\beta - f_s\sin\beta - mg = 0 \quad (2)$$

$$N\sin\beta + \mu_s N\cos\beta = \frac{m\omega^2 R^2}{R} \quad (1A)$$

$$N\cos\beta - \mu_s N\sin\beta = mg \quad (1B)$$

divide $\dfrac{1A}{1B} = \dfrac{N(\sin\beta + \mu_s \cos\beta)}{N(\cos\beta - \mu_s \sin\beta)} = \dfrac{R\omega^3}{g}$

$$\omega = \sqrt{\frac{g}{R}\frac{(\sin\beta + \mu_s \cos\beta)}{(\cos\beta - \mu_s \sin\beta)}}$$



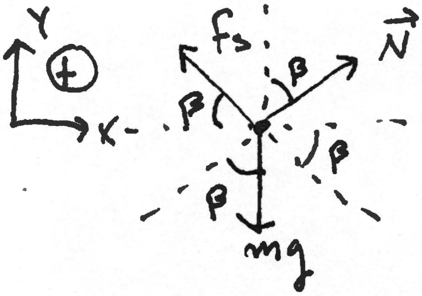
$\tan\beta = \dfrac{h}{R}$

$R = \dfrac{h}{\tan\beta}$

$$\omega = \sqrt{\frac{g\tan\beta}{h}\frac{(\sin\beta + \mu_s \cos\beta)}{(\cos\beta - \mu_s \sin\beta)}}$$

$$\boxed{T_{max} = 2\pi\sqrt{\frac{h(\cos\beta - \mu_s \sin\beta)}{g\tan\beta(\sin\beta + \mu_s \cos\beta)}}}$$

(b) For μ_{min} the mass m will want to slide down the cone



$$\vec{N} = N \sin\beta \,\hat{x} + N \cos\beta \,\hat{y}$$

$$\vec{F_s} = -f_s \cos\beta \,\hat{x} + f_s \sin\beta \,\hat{y}$$

$$T = \frac{2\pi}{\omega} \qquad V = R\omega$$

$$f_s = \mu_s N$$

$$\sum F_x = N \sin\beta - f_s \cos\beta = \frac{mV^2}{R} = \frac{m R^2 \omega^2}{R} = mR\omega^2$$

$$\sum F_y = N \cos\beta + f_s \sin\beta - mg = 0$$

$$N \sin\beta - \mu_s N \cos\beta = mR\omega^2 \qquad (1)$$

$$N \cos\beta + \mu_s N \sin\beta = mg \qquad (2)$$

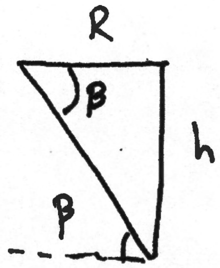
$$\frac{(1)}{(2)} = \frac{\cancel{N}(\sin\beta - \mu_s \cos\beta)}{\cancel{N}(\cos\beta + \mu_s \sin\beta)} = \frac{\cancel{m}R\omega^2}{\cancel{m}g}$$

$$\omega = \sqrt{\frac{g}{R} \, \frac{\sin\beta - \mu_s \cos\beta}{\cos\beta + \mu_s \sin\beta}}$$

Yildiz MT1

Fall 2019

4



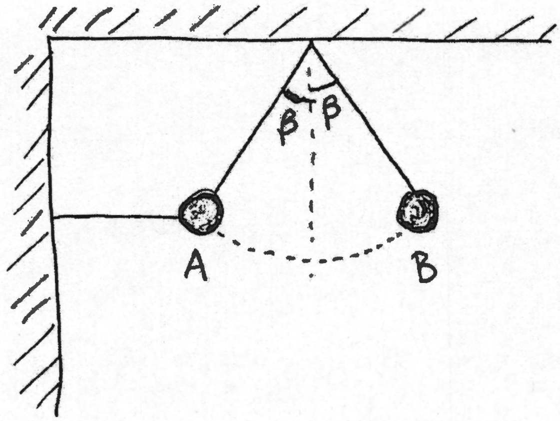
$$\tan\beta = \frac{h}{R}$$

$$R = \frac{h}{\tan\beta}$$

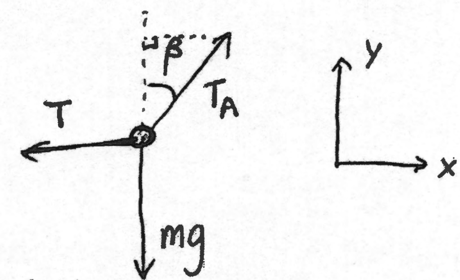
$$\omega = \sqrt{\frac{g\tan\beta}{h}\,\frac{\sin\beta - \mu_s\cos\beta}{\cos\beta + \mu_s\sin\beta}}$$

$$\boxed{T_{min} = 2\pi\sqrt{\frac{h(\cos\beta + \mu_s\sin\beta)}{g\tan\beta(\sin\beta - \mu_s\cos\beta)}}}$$

4



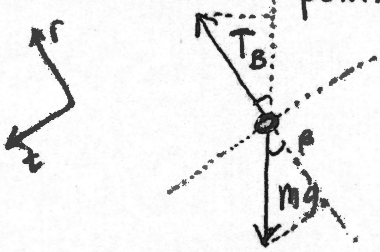
At point A the forces are:



These all balance:

$$\Rightarrow \begin{cases} T_A \sin\beta - T = 0 & (x\text{-eq}) \\ T_A \cos\beta - mg = 0 & (y\text{-eq}) \end{cases}$$

The forces at point B are:



These balance in the radial direction, but not in the tangential direction:

$$\Rightarrow \begin{cases} T_B - mg\cos\beta = 0 & (\text{radial eq}) \\ mg\sin\beta = ma_t & (\text{tangential eq}) \end{cases}$$

Take the y-eq. from configuration A and the radial equation from configuration B:

$$\begin{cases} T_B = mg\cos\beta \\ T_A = \dfrac{mg}{\cos\beta} \end{cases} \Longrightarrow \boxed{\frac{T_B}{T_A} = \cos^2\beta}$$

Note*: We are able to set the radial acceleration to zero because:

1) The motion of the mass is confined to a circle, so $a_r = V^2/R$
2) At point B, $v = 0$.

Problem 5 solution

Fall 2019 Physics 7A Lec 002 (Yildiz) Midterm I

The passenger is moving in a circle of radius $r = R_0 + L\sin\theta$ and therefore has centripetal acceleration $a_c = v^2/r$. The only forces acting on m are tension (F_T) and gravity (mg). We apply Newton's second law for the horizontal and vertical components.

$$\sum F_x = F_T \sin\theta = ma_c \tag{1}$$
$$\sum F_y = F_T \cos\theta - mg = 0 \tag{2}$$

a) The second equation can be used to solve for tension ($F_T = mg/\cos\theta$). We can then obtain an expression for v by plugging in values for F_T, a_c and r.

$$\frac{\cancel{m}g}{\cos\theta}\sin\theta = \cancel{m}\frac{v^2}{r} \tag{3}$$
$$g\tan\theta = \frac{v^2}{R_0 + L\sin\theta} \tag{4}$$

$$\boxed{v = \sqrt{(g\tan\theta)(R_0 + L\sin\theta)}} \tag{5}$$

b) We have already obtained an expression for F_T

$$\boxed{F_T = \frac{mg}{\cos\theta}} \tag{6}$$

c) We use the relation between circumference, T and v, plug in expressions for v and r, and then solve for T.

$$v = \frac{2\pi r}{T} \implies T = 2\pi\frac{r}{v} \tag{7}$$

$$T = 2\pi\frac{R_0 + L\sin\theta}{\sqrt{(g\tan\theta)(R_0 + L\sin\theta)}} \tag{8}$$

$$\boxed{T = 2\pi\sqrt{\frac{R_0 + L\sin\theta}{g\tan\theta}}} \tag{9}$$