S20 PHYSICS 7B: Wang MT 1 Solutions

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1 Problem 1

(a)

The average *translational* kinetic energy is given by the equipartition theorem, applied only to the translational degrees of freedom:

$$\langle KE \rangle = \frac{3}{2}kT. \tag{1.1}$$

The answer is therefore still 300 K.

(b)

The RMS speed is given by equating the above to the usual expression for kinetic energy:

$$\frac{3}{2}kT = \frac{1}{2}m\langle v^2 \rangle \implies v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}. \tag{1.2}$$

An oxygen molecule at temperature T_O therefore has RMS speed $v_{O,\text{rms}} = \sqrt{\frac{3kT_O}{32u}}$ while a helium atom has at temperature T_H has RMS speed $v_{H,\text{rms}} = \sqrt{\frac{3kT_H}{4u}}$. Setting the two equal gives:

$$T_H = \frac{4}{32}T_O = \frac{300}{8} = 37.5 \text{ K}. \tag{1.3}$$

(c)

At 300 K, the molecule receives contributions from 3 translational and 2 rotational degrees of freedom, but the vibrational modes are frozen out. Therefore $d = 5$ and $C_V = \frac{5}{2}R$. At 3000 K, the 2 vibrational modes activate, giving $d = 7$, so $C_V = \frac{7}{2}$ in this case.

2 Problem 2

(a)

The change in energy ΔE_{ac} can be computed from the given information, using the fact that E is a state function, so:

$$\Delta E_{ac} = Q_{ac} - W_{ac} = Q_{abc} - W_{abc} = \Delta E_{abc}. \tag{2.1}$$

Then we can solve for Q_{abc} as:

$$Q_{abc} = \Delta E_{abc} + W_{abc} = Q_{ac} - W_{ac} + W_{abc} = -65 \text{ J} + 32 \text{ J} - 54 \text{ J} = -87 \text{ J}. \tag{2.2}$$

(b)

We can see that the path $c \to d$ is simply $a \to b$ in reverse, but at pressure P_c. In other words:

$$W_{ab} = P_b \Delta V_{ab} \quad \implies \quad W_{cd} = P_c \Delta V_{ba} = -P_c \Delta V_{ab}. \tag{2.3}$$

Plugging in for P_c gives

$$W_{cd} = -\frac{1}{2} P_b \Delta V_{ab} = -\frac{1}{2} W_{ab} = -\frac{1}{2} W_{abc} = 27 \text{ J}. \tag{2.4}$$

Then because $W_{da} = 0$, we have $W_{cda} = 27$ J.

(c)

We know that $\Delta E_{cda} = -\Delta E_{abc} = -\Delta E_{ac}$. Therefore, we must have:

$$Q_{cda} = \Delta_{cda} + W_{cda} = -\Delta E_{ac} + W_{cda} = -Q_{ac} + W_{ac} + W_{cda}. \tag{2.5}$$

Plugging in numbers:

$$Q_{cda} = 65 \text{ J} - 32 \text{ J} + 27 \text{ J} = 60 \text{ J}. \tag{2.6}$$

(d)

We calculated in the previous parts:

$$\Delta E_{ac} = Q_{ac} - W_{ac} = -33 \text{ J}. \tag{2.7}$$

Therefore,

$$\Delta E_{ca} = -\Delta E_{ac} = 33 \text{ J}. \tag{2.8}$$

(e)

We must have

$$\Delta E_{cda} = \Delta E_{cd} + \Delta E_{da} = (E_{\text{int},d} - E_{\text{int},c}) + (E_{\text{int},a} - E_{\text{int},d}) = \Delta E_{ca}. \tag{2.9}$$

Therefore,

$$\Delta E_{da} = \Delta E_{ca} - \Delta E_{cd} = Q_{da} - W_{da}, \tag{2.10}$$

but we know $W_{da} = 0$, so:

$$Q_{da} = \Delta E_{ca} - \Delta E_{cd} = 33 \text{ J} - 12 \text{ J} = 21 \text{ J}. \tag{2.11}$$

3 Problem 3

(a)

We predict that the final state of the mixture will be a solid aluminum cup and liquid water. The calorimetry equation is then:

$$m_w L_f + m_w c_w (T_f - T_{i,w}) + m_a c_a (T_f - T_{i,a}) = 0, \tag{3.1}$$

where m_f is the mass of the water, L_f is the latent heat of fusion for the water, c_w is the specific heat of water, $T_{i,w}$ is the initial temperature of the liquid water, m_a is the mass of the cup, c_a is the specific heat of aluminum, $T_{i,a}$ is the initial temperature of the aluminum cup, and T_f is the final temperature. Solving for T_f:

$$T_f = \frac{m_a c_a T_{i,a} - m_w L_f + m_w c_w T_{i,w}}{m_w c_w + m_a c_a}. \tag{3.2}$$

Plugging in numbers:

$$T_f = \frac{(400\text{ g})(0.9\text{ J/g.K})(353\text{ K}) - (20\text{ g})(333\text{ J/g}) + (20\text{ g})(4.2\text{ J/g.K})(273\text{ K})}{(20\text{ g})(4.2\text{ J/g.K}) + (400\text{ g})(0.9\text{ J/g.K})} \approx 323\text{ K}. \tag{3.3}$$

(b)

The entropy change is:

$$\Delta S = \int \frac{dQ}{T} = \frac{m_w L_f}{T_{i,w}} + m_w c_w \log \frac{T_f}{T_{i,w}} + m_a c_a \log \frac{T_f}{T_{i,a}} \tag{3.4}$$

4 Problem 4

Let us set the 4 charges in the xy-plane and put the charge Q along the z-axis. It is located a distance $r = \sqrt{b^2 + d^2/2}$ from each corner of the square. By symmetry, the electric field due to the charges in the square points along the z-axis (any component pointing along the x and y directions will cancel out). We therefore just need to determine the magnitude of the electric field pointing in the z-direction, corresponding to the magnitude of the z-component of each electric field.

We can determine it with a little trigonometry: we want to multiply the field vector magnitude by $\sin\theta$, θ is the angle between the xy-plane and each electric field vector. But we also know that $\sin\theta = \frac{b}{r} = \frac{b}{\sqrt{b^2+d^2/2}}$, so:

$$E_z = |E|\sin\theta = \frac{kq}{r^2}\sin\theta = \frac{kqb}{(b^2 + d^2/2)^{3/2}}. \tag{4.1}$$

Multiplying by 4 to get the total field at that point gives the final force as:

$$\vec{F} = Q\vec{E}_{\text{net}} = 4QE_z\hat{z} = \frac{4kqQb}{(b^2 + d^2/2)^{3/2}}\hat{z}. \tag{4.2}$$

5 Problem 5

(a)

The coefficient of performance of a heat pump is

$$C = \frac{Q}{W}, \tag{5.1}$$

where Q is the heat moved, and W is the work the pump does. We are heating, so $Q = Q_H$ is the heat moved from the cold reservoir to the hot reservoir. We therefore have:

$$C = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}, \tag{5.2}$$

where Q_C is the heat exhausted to the cold reservoir. Because the heat pump is ideal, we have

$$C = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}. \tag{5.3}$$

If the heat pump is maintaining a constant temperature, the rate of heat $\frac{dQ_H}{dt}$ being moved needs to precisely equal the heat leaving the house through conduction:

$$\frac{dQ_H}{dt} = \frac{dW}{dt}C = 1300\frac{T_H}{T_H - T_C} = 650(T_H - T_C). \tag{5.4}$$

We want to determine T_C, so:

$$2T_H = (T_H - T_C)^2 \quad \Longrightarrow \quad T_C^2 - 2T_H T_C + T_H^2 - 2T_H = 0, \tag{5.5}$$

which we can solve as

$$T_C = T_H - \sqrt{2T_H}. \tag{5.6}$$

Plugging in $T_H = 294$ K gives

$$T_C \sim 270 \text{ K}. \tag{5.7}$$

(b)

If the heat pump runs p percent of the time, the effective work is $pW = 1300p$. Using our equation above gives:

$$1300p\frac{T_H}{T_H - T_C} = 650(T_H - T_C). \tag{5.8}$$

Solving for p:

$$p = \frac{(T_H - T_C)^2}{2T_H}, \tag{5.9}$$

and plugging in the given temperatures:

$$p = \frac{(294 - 282)^2}{2(294)} = \frac{144}{294} = \frac{12}{49} \sim 24\%. \tag{5.10}$$