PHYSICS 7B, Lecture 1 – Spring 2020
Midterm 1, C. Bordel
Monday, February 24th
7pm-9pm

- Student name:
- Student ID #:
- Discussion section #:
- Name of your GSI:
- Day/time of your DS:

Make sure you show all your work and justify your answers in order to get full credit!

Problem 1 – Expanding gas container (20 pts)

n moles of water vapor (H_2O), considered as an ideal gas, are enclosed in a container. The initial volume and temperature of the container are V_i and T_i respectively. The container, made of a thin shell of an unknown isotropic material, is heated up until its volume increases by 10%, and the final pressure of the gas is the same as the initial pressure. You may assume that the gas is in thermodynamic equilibrium with the container before and after the expansion, and that the vibrational degrees of freedom of the gas are frozen in this temperature range.

a) Determine the final temperature T_f of the gas as a function of T_i exclusively.

b) Determine α, the linear coefficient of thermal expansion of the container.

c) Calculate the ratio of the *rms* speed of gas molecules at temperature T_f over their *rms* speed at temperature T_i. Explain why your result is greater or smaller than 1.

d) Compare the change in internal energy between T_i and T_f of n moles of neon (Ne), a monatomic noble gas, whose molecular mass is almost the same as that of water molecules, with the change in internal energy of n moles of water vapor. Explain your result.

<u>Problem 2</u> – An unusual thermodynamic process (20 pts)

n moles of a monatomic ideal gas undergo a non-traditional reversible thermodynamic process from pressure and volume (P_A, V_A) to pressure $P_B=2P_A$, following the curve $PT=const$.

a) Calculate volume V_B and sketch the $A \to B$ process on a P-V diagram.

b) Calculate the work done by the gas from A to B, and represent it graphically on the P-V diagram. Explain the sign based on the volume change.

c) Calculate the heat absorbed by the gas from A to B.

d) Calculate the change in entropy of the gas from A to B. Does your result violate the second law of thermodynamics?

Problem 3 – Phase changes & Point charges (20 pts)

The two parts below are totally independent.

Part 1

A material follows the phase diagram represented in Fig.1. A solid block of the substance, initially at atmospheric pressure and temperature -100ºC, is placed at room temperature under the same pressure.

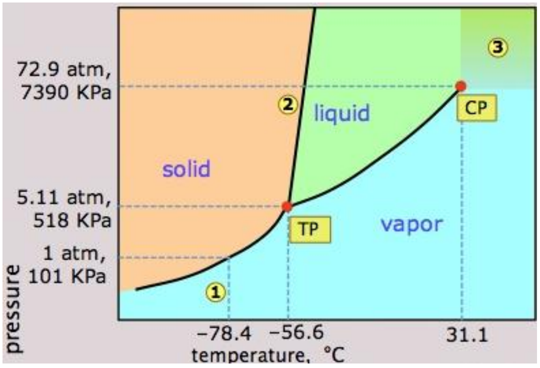

Figure 1

a) Based on the phase diagram shown in Fig.1, explain in which phase(s) the material will be when it reaches thermodynamic equilibrium with its environment.

b) Sketch the variation of the material's temperature as a function of time and explain the various sections of the plot as thoroughly as possible. *You may assume that the rate of heat flow is constant.*

Part 2

Let's consider a discrete distribution of four point-charges located as follows in an (x,y) coordinate system: q at point $A(d,0)$, $-q$ at point $B(0,d)$, $-q$ at point $C(-d,0)$ and $2q$ at point O $(0,0)$. Electric charge q is positive and d is a length. You may use $(\hat{\imath}, \hat{\jmath})$ as the unit vectors associated with this cartesian coordinate system.

c) Draw, qualitatively, all the individual electrostatic forces acting on charge $2q$, as well as their resultant.

d) Calculate the net electrostatic force created by the charges located at points A, B and C and acting on charge $2q$, and determine the electric field generated at point O by the same three point charges.

<u>Problem 4</u> – Heat engine (20 pts)

A heat engine operates under the following reversible cycle:
$A \rightarrow B$: adiabatic expansion up to volume $V_B = 4V_A$
$B \rightarrow C$: isobaric compression down to $V_C = 2V_A$
$C \rightarrow D$: adiabatic compression down to $V_D = V_A$
$D \rightarrow A$: isochoric warming
The engine uses n moles of a working substance that can be considered as an ideal two-dimensional monatomic gas with 2 degrees of freedom. Express all your answers in terms of V_A , P_A , n and R. You may assume that the vibrational degrees of freedom of the gas are not activated in the temperature range in which the engine operates.

a) Draw the corresponding *(ABCD)* cycle on a *P-V* diagram and determine all the temperatures T_A, T_B, T_C and T_D.

b) Calculate the net work produced by the engine over a full cycle.

c) Calculate the efficiency of the engine.

d) Calculate the efficiency of the Carnot engine operating between the same two extreme temperatures and compare with the efficiency found in part *(c)*.

Problem 5– A rod as a thermal bridge (20 pts)

A container contains a mixture of ice and water of total mass m at freezing temperature T_F. Another container contains the same mass m of boiling water at temperature T_B and the source of heat is kept on. A slightly conical rod of thermal conductivity k and total length L is placed between the two containers and piercing them on each side, as shown in Fig.2. The rod has a linear increase in radius, from R_1 in the ice-water mixture to R_2 ($R_2 > R_1$) in the boiling water. Additionally, the rod is well-insulated so that no heat is lost out the sides.

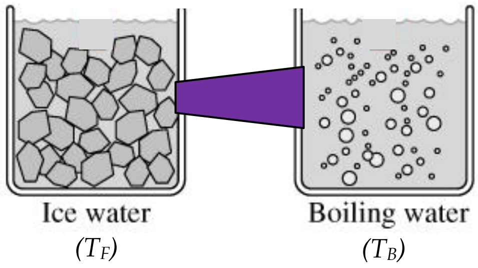

Figure 2

a) Determine the rate of conductive heat flow through the rod and explain the direction of the heat flow.

After a while, all the ice of the mixture has melted but the temperature has not started to rise yet. In the other container, only half of the initial mass of water is left due to vaporization when the heat source is turned off. You may assume that evaporation was negligible in both containers. All the water, of specific heat C_w, is now poured in a single thermally insulated container. *Keep your answers in symbolic form.*

b) What is the equilibrium temperature reached by the system?

c) Determine the change in entropy of the system {water + surroundings}.

$$\Delta l = \alpha l_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$PV = NkT = nRT$$

$$\frac{1}{2} m\overline{v^2} = \frac{3}{2} kT$$

$$E_{int} = \frac{d}{2} NkT$$

$$Q = mc\Delta T = nC\Delta T$$

$$Q = mL \text{ (For a phase transition)}$$

$$\Delta E_{\text{int}} = Q - W$$

$$dE_{\text{int}} = dQ - PdV$$

$$W = \int PdV$$

$$C_P - C_V = R = N_A k$$

$$PV^\gamma = \text{const. (For a reversible adiabatic process)}$$

$$\gamma = \frac{C_P}{C_V} = \frac{d+2}{d}$$

$$C_V = \frac{d}{2} R$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$e = \frac{W_{\text{net}}}{Q_{\text{in}}}$$

$$e_{\text{ideal}} = 1 - \frac{T_L}{T_H}$$

$$S = \int \frac{dQ}{T} \text{ (For reversible processes)}$$

$$dQ = TdS$$

$$\Delta S_{syst} + \Delta S_{env} > 0 \text{ (For irreversible processes)}$$

$$\oint dE = \oint dS = 0$$

	Q	W
Isobaric	$C_P n\Delta T$	$P\Delta V$
Isochoric	$C_V n\Delta T$	0
Isothermal	$nRT\ln\left(\frac{V_f}{V_0}\right)$	$nRT\ln\left(\frac{V_f}{V_0}\right)$
Adiabatic	0	$-\frac{d}{2}(P_f V_f - P_0 V_0)$

$$\vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r} = \frac{kQ_1Q_2}{r^2}\hat{r}$$

$$\vec{F} = Q\vec{E}$$

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 r^2}\hat{r} = \frac{k\,dQ}{r^2}\hat{r}$$

$$\int (1+x^2)^{-1/2} dx = \ln(x + \sqrt{1+x^2})$$

$$\int (1+x^2)^{-1} dx = \arctan(x)$$

$$\int (1+x^2)^{-3/2} dx = \frac{x}{\sqrt{1+x^2}}$$

$$\int \frac{x}{1+x^2} dx = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{1}{\cos(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|\right)$$

$$\int \frac{1}{\sin(x)} dx = \ln\left(\left|\tan\left(\frac{x}{2}\right)\right|\right)$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$(1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha-1)\alpha}{2}x^2$$

$$\ln(1+x) \approx x - \frac{x^2}{2}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$1 + \cot^2(x) = \csc^2(x)$$

$$1 + \tan^2(x) = \sec^2(x)$$