UNIVERSITY OF CALIFORNIA
Department of Materials Science and Engineering

Fall 2019 Professor Ritchie

MSE 113
Mechanical Behavior of Engineering Materials

Midterm Exam #2
October 31st, 2019

Name: ______________________________

SID#: ______________________________

Problem	Total	Score
1	30	
2	30	
3	40	

Problem 1

The below true stress-true plastic strain relation describes the behavior of a quenched and tempered SAE 4340 steel (E = 207 GPa)

$$\sigma_T = 2.76\, \varepsilon_T^{0.1}\, GPa$$

Where σ_T and ε_T are true stress and true strain respectively. The true plastic strain at fracture for this metal was measured as 0.45. Determine the following:

a) True fracture strength, σ_f
b) Total true strain at fracture
c) Strain hardening exponent, n
d) Strength at the 0.2% 'offset' yield strength (as seen in Fig 2.26 in textbook)
e) Reduction in area %RA at fracture
f) True fracture ductility, ε_f
g) True strain at the ultimate strength
h) Engineering ultimate strength, S_{US}

Problem 1 Solution:

Given: E = 207 GPa, $\sigma_T = 2.76\, \varepsilon_T^{0.1}\, GPa$, $\varepsilon_{P,T}$ = 0.45

a) $\sigma_f = 2.76\,(0.45)^{0.1}\, GPa$ => $\boldsymbol{\sigma_f}$ **= 2.55 Gpa**
b) $\varepsilon_{total} = \varepsilon_{elastic} + \varepsilon_{plastic} = \frac{\sigma_f}{E} + 0.45 = \frac{2.55\, GPa}{207\, GPa} + 0.45$ => $\boldsymbol{\varepsilon_{total}}$ **= 0.462**
c) ***n*** **= 0.1**
d) $\sigma_{yield} = 2.76\,(0.002)^{0.1}\, GPa$ => $\boldsymbol{\sigma_{yield}}$ **= 1.48 Gpa**

Or

One can find the intercept between a line parallel to the elastic region but offset and the plastic zone (a little more correct; both fine)

$$\sigma_T = 2.76\, \varepsilon_T^{0.1} = (\varepsilon_T - 0.002)\, E \;\; => \varepsilon_T = 0.0104$$

=> $\boldsymbol{\sigma_{yield}}$ **= 1.75 Gpa**

e) To determine %RA, need to find engineering strain $\varepsilon_e$from the true plastic strain at fracture and can then find assuming constant volume

$$\varepsilon_T = \ln(1+\varepsilon_e) \quad => \quad \varepsilon_e = e^{\varepsilon_T} - 1 = e^{0.45} - 1 = 0.568$$

From constant volume, we can derive the relation $\frac{A_o}{A} = 1 + \varepsilon_e$

$$\%RA = \frac{A_o - A}{A_o} = 1 - \frac{A}{A_o} = 1 - \frac{1}{1 + \varepsilon_e}$$ => **%RA = 36%**

f) $\varepsilon_f = \varepsilon_{P,T}$ => $\boldsymbol{\varepsilon_f}$ **= 0.45**
g) As the sample necks, the stress experienced remains ~ constant => dP = 0

$$\sigma_T = \frac{P}{A} \quad => \quad \sigma\, dA + A\, d\sigma = 0 \quad => \quad \frac{d\sigma}{\sigma} = -\frac{dA}{A}$$

And due to constant volume: $d\varepsilon_T = \frac{dl}{l} = -\frac{dA}{A} = \frac{d\sigma}{\sigma}$ => $\frac{d\sigma}{d\varepsilon} = \sigma$

Using this relationship: $k\, n\, \varepsilon^{n-1} = k\, \varepsilon^{n}$ => $\boldsymbol{\varepsilon = n = 0.1}$

h) Using the value for strain at the ultimate strength, we can get the stress at the ultimate strength

$$\sigma_{US} = 2.76\,(0.1)^{0.1}\, GPa = 2.19 \text{ GPa}$$

Relating these 'true' values to engineering stress S_{US}: $\sigma_T = S_e(1+\varepsilon_e) = S_e(e^{\varepsilon_T})$

$$S_{US} = \frac{\sigma_{US}}{e^{\varepsilon_{US}}} = \frac{2.19 \text{ GPa}}{e^{0.1}} = 1982 \text{ MPa}$$

Problem 2 question and answer:

a. For each of the following metals strengthening techniques, **state in 4 sentences or less:**

1. The microstructural change that causes the strengthening
2. The physical mechanism of deformation that is affected by the microstructural change and why it's affected
3. How one processes the metal to achieve the strengthening technique
4. Any limitations/restrictions to the technique, if any exist. (For example, are there systems that the technique will *not* work for? Are there possible environmental factors that change the effectiveness of the technique…? *etc.*)

Precipitation hardening:

When the solute concentration in an alloy exceeds the limits of solubility for the matrix phase, equilibrium conditions dictate the nucleation and growth of second-phase particles, provided that suitable thermal conditions are present. Through holding a material at a specific temperature, precipitation of β particle either within α grains or at grain boundaries occurs. This increases the difficulty of dislocation motion through the lattice. The particle size and shape are functions of temperature and time, and there is a peak aging time that gives maximum strength to precipitates. After this time, the precipitates begin to lose strength. The stress required for dislocations to loop around precipitates is a function of precipitate particle size. The looping is controlled by the spacing between particles. Aging also leads to a loss of coherency, decreasing strength. This can be considered intrinsic (it is, after all, a process which has no external contributions).

Solid-solution strengthening:

Solid-solution strengthening combines two or more elements such that a single-phase microstructure is retained. This causes various elastic, electrical, and chemical interactions to take place between stress fields of the solute atoms and dislocations present in the lattice. We know that shear stresses are associated with screw dislocations, whereas both shear and hydrostatic stress fields are involved with edge dislocations. If nonsymmetrical interactions take place (i.e. octahedral interstitial sites in BCC lattices, which interaction with nonsymmetrical stress components of edge and screw dislocations), large amounts of strengthening can occur. This takes place in carbon steels. for example, octahedral interstitial sites are symmetric in FCC lattices, and carbon atoms have space to sit in those interstitial sites, so austenite will not be strengthened much by carbon inclusion. However, the addition of carbon to a BCC lattice causes the development of a body-centered tetragonal lattice of martensite. In addition to the interstitial solution, substitutional defects (such as Pd or Pt in Cu), can also take place. This is extrinsic, as other materials are added.

Cold working (to cause strain/work hardening):

Strain hardening, or work hardening, occurs when a metal is strained beyond its yield point. An increasing stress is required to produce additional plastic deformation and the metal becomes stronger and more difficult to deform. The strengthening occurs because of dislocation movement and generation within the crystal structure of the material. As a material is work hardened it becomes increasingly saturated with new dislocations, and more dislocations are prevented from nucleating, so a resistance to formation develops. This resistance to dislocation formation manifests itself as a resistance to plastic deformation, hence the observed strengthening. This is intrinsic, as it is characterized by manipulation of the material.

Grain size reduction:

The Hall-Petch equation relates the yield strength of a polycrystalline material by the equation $\sigma_{ys} = \sigma_i + k_y d^{-1/2}$. Grain size refinement can result, therefore, an improved yield strength and toughness. The refinement techniques provide barriers to dislocation movement. The universal use of the Hall-Petch relation is not recommended, as other equations can better describe the strength-grain size relation. In addition, there is obviously a minimum grain size for which the relation applies. This is intrinsic, as grain size can be refined without any additions.

b. What is the difference between strength, energetic toughness, and stiffness?

Strength = ability to withstand deformation ->
yield = before plastic deformation (usually defined as 0.2% offset);
ultimate = onset of necking (maximum of an engineering stress/strain curve (dP=0))
Fracture strength = stress at break (last stress on an engineering stress/strain curve)

Energetic toughness – ability of a material to absorb energy. Estimated by area under engineering stress/strain curve; can be found w/ a Charpy test

Stiffness – ability to resist elastic deformation when a stress is applied. Typically defined as E (Young's modulus), the slope of the linear portion of the stress/strain curve (either engineering or true -> small strains means they're basically the same)

c. Why are all the slip systems in an FCC metal of the form {111} <110>?

Dislocations move on close-packed plains in close-packed directions -> {111} family and <110> family are close-packed in an FCC structure.

Problem 3

You are in the process of designing an AISI 304 austenitic stainless-steel rod that will be used in an application that will require it to withstand temperatures of 1100 F constantly during continuous operation. However, creep is a concern. Assuming that the experimental data in the given temperature range follows a constitutive law:

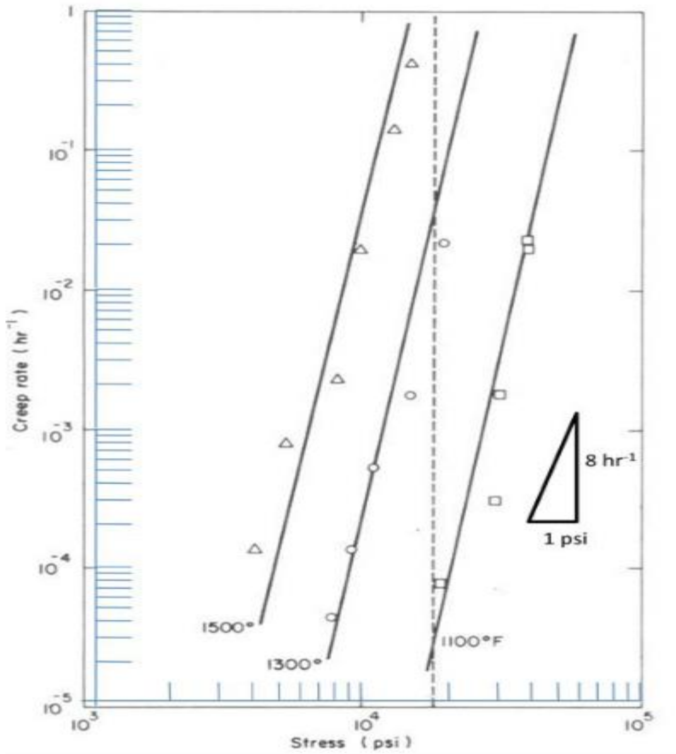
$$\frac{\dot{\varepsilon}_{ss}}{\dot{\varepsilon}_0} = \left(\frac{\sigma_{11}}{\sigma_0}\right)^m \exp\left(-\frac{H}{kT}\right)$$

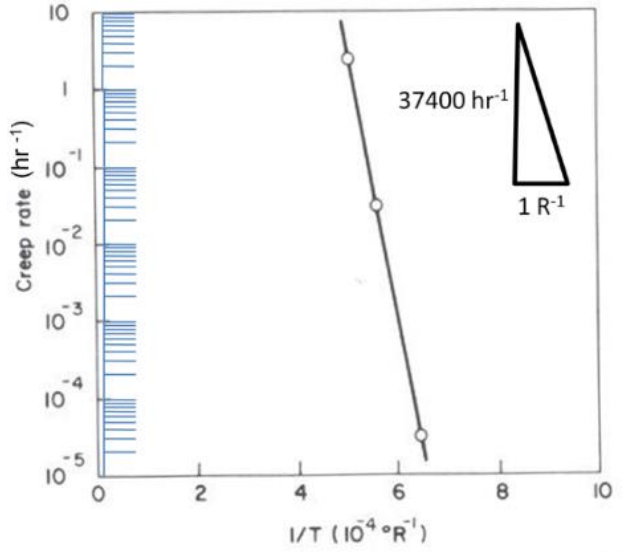
[where *H* is the activation energy, *T* is the temperature in Rankins, and *k* is Boltzmann's constant, and *m* is the creep exponent]. **Note: Non-SI units are fine and expected**

a) Determine m, H, and σ_0 from the experimental data (below on next page). To save you all some work, we have taken data from the first plot to generate the accompanying creep rate vs. 1/T curve. Slopes are as indicated. For simplicity, assume $\dot{\varepsilon}_0 = 1\ hr^{-1}$
b) Another laboratory tests your material and finds the primary creep is responsible for an observed 0.76% strain and the material has elastic properties *E*= 29 Mpsi at room temperature and E = 22.3 Mpsi at 1100 F.

   You machine a cylindrical rod of diameter 0.25 in and load it in tension with a stress of 10,000 psi at 1100 F. What is the lifetime of the rod under these conditions if it is to be decommissioned upon reaching a total strain of 1.5%?

   Be clear about how you are handling the question of elastic, primary, and secondary creep strains.





Problem 3 Solution:

a) First, assuming all the lines are parallel and given the constitutive law

$$\frac{\dot{\varepsilon}_{ss}}{\dot{\varepsilon}_0} = \left(\frac{\sigma_{11}}{\sigma_0}\right)^m \exp\left(-\frac{H}{kT}\right)$$

To Determine H

We can begin by fixing σ_{11} while varying T. While this can all be determined from the full-data graph, it's more apparent on the strain rate vs. 1/T plot. Looking at this plot and the given relationship, we can see the slope of the 1/T plot should = -*H*/*k*. Note that the plot is a $\log_{10}$ plot, so you need to convert.

$$slope = \frac{\Delta \log\left(\dot{\varepsilon}_{ss}/\dot{\varepsilon}_0\right)}{\Delta(1/\mathrm{T})} = -37400 = -\frac{H}{k\,(\ln 10)}$$

⇨ $H = 37400(6.79 \times 10^{-23}) \ln 10 = \mathbf{5.85 \times 10^{-18}\ in \cdot lbf} = \mathbf{6.61 \times 10^{-18}\ J}$

To Determine m

If we fix T while varying stress (the opposite of the prior part), then you can use this information to determine *m* from the strain rate vs. stress graph. The slope of these parallel lines = *m* (nice and straightforward)

$$slope = m = \frac{\Delta \log(\dot{\varepsilon}_{ss}/\dot{\varepsilon}_0)}{\Delta \log(\sigma_{11}/\sigma_0)} = \mathbf{8}$$

To Determine σ_0

Just choose a point on one of the strain rate vs. stress line and plug in all the values you now have to get σ_0. Let $\dot{\varepsilon}_0$ = 1 hr^{-1}

Ex. For (10^4 psi, 2 x 10^{-4} hr^{-1}) on the T = 1300 F line:

$$\frac{2\ x\ 10^4\ hr^{-1}}{1\ hr^{-1}} = \left(\frac{104\ \text{psi}}{\sigma_0}\right)^8 \exp\left(-\frac{5.85\times10^{-18}\ \text{in}\cdot\text{lbf}}{(5.67x\ 10^{-24}\ ft * lb/R)(1300\ F)}\right)$$

⇨ $\boldsymbol{\sigma_0}$ **= 64 psi = 441 kPa** [Note: may have a little variation due to accuracy of point chosen]

$$\frac{\dot{\varepsilon}_{ss}}{\dot{\varepsilon}_0} = \left(\frac{\sigma_{11}}{64\ \text{psi}}\right)^8 \exp\left(-\frac{5.85\times10^{-18}\ \text{in}\cdot\text{lbf}}{kT}\right)$$

b) Begin by looking at the contribution of elastic strain. This one's pretty straightforward, using the Young's modulus at the temp. of interest (1100 F)

$$\sigma = E\varepsilon \qquad => \qquad \varepsilon_{el} = \frac{\sigma}{E_{1100F}} = \frac{10{,}000\ psi}{22.3\ Mpsi} = 4.484\text{ x }10^{-4}$$

We were already given the total strain (our end condition of 0.015), and the contribution of the primary strain, so we next need to use these to find the secondary strain (and thus the lifetime)

$$\varepsilon_{Tot} = \varepsilon_{el} + \varepsilon_I + \dot{\varepsilon}_{ss}t + \varepsilon_{III}$$
$$0.015 = 4.484\text{ x }10^{-4} + 0.0076 + \dot{\varepsilon}_{ss}t$$
$$\dot{\varepsilon}_{ss}t = 6.952\text{ x }10^{-3}$$

Now the final step before finding t is to determine $\dot{\varepsilon}_{ss}$ at 1100 F and 10,000 psi

$$\dot{\varepsilon}_{ss} = (1\ hr^{-1})\left(\frac{10{,}000\ psi}{64\text{ psi}}\right)^8 1.022\text{ x }10^{-24} = 3.631\text{ x }10^{-7}\ hr^{-1}$$

And now just plug in to find lifetime

$$t = \frac{6.952\text{ x }10^{-3}}{3.631\text{ x }10-7} = \mathbf{19150\ hr}$$