UNIVERSITY OF CALIFORNIA
Department of Materials Science and Engineering

Fall 2019 | Professor Ritchie

MSE 113
Mechanical Behavior of Engineering Materials

Midterm Exam #2
October 31st, 2019

Name: ______________________________

SID#: ______________________________

Problem	Total	Score
1	30	
2	40	
3	30	

Problem 1

The below true stress-true plastic strain relation describes the plasticity behavior of a quenched and tempered SAE 4340 steel (E = 207 GPa)

$$\sigma_T = 2.76\, \varepsilon_T^{0.1}\ GPa$$

where σ_T and ε_T are true stress and true strain respectively. The true plastic strain at fracture for this metal was measured as 0.45. Determine the following:

a) True fracture strength, σ_f
b) Total true strain at fracture
c) Strain hardening exponent, n
d) Strength at the 0.2% 'offset' yield strength (as seen in Fig 2.26 in textbook)
e) Reduction in area %RA at fracture
f) True fracture ductility, ε_f
g) True strain at necking
h) Engineering ultimate strength, S_{US}

For all calculations, state all your assumptions.

Problem 2

a. For each of the following metals strengthening techniques, **state in 4 sentences or less:**

1. The microstructural change that causes the strengthening
2. The physical mechanism of deformation that is affected by the microstructural change and why it's affected
3. How one processes the metal to achieve the strengthening technique
4. Any limitations/restrictions to the technique, if any exist. (For example, are there systems that the technique will *not* work for? Are there possible environmental factors that change the effectiveness of the technique…? *etc.*)

Precipitation hardening:

Solid-solution strengthening:

Cold working (to cause strain/work hardening):

Grain size reduction:

b. What is the difference between strength, energetic toughness, and stiffness?

c. Why are all the slip systems in an FCC metal of the form {111} <110>?

Problem 3

You are in the process of designing an AISI 304 austenitic stainless-steel rod that will be used in an application that will require it to withstand temperatures of 1100 F constantly during continuous operation. However, creep is a concern. Assuming that the experimental data in the given temperature range follows a constitutive law:

$$\frac{\dot{\varepsilon}_{ss}}{\dot{\varepsilon}_0} = \left(\frac{\sigma_{11}}{\sigma_0}\right)^m \exp\left(-\frac{H}{kT}\right)$$

[where *H* is the activation energy, *T* is the temperature in Rankins, and *k* is Boltzmann's constant, and *m* is the creep exponent]. **Note: Non-SI units are fine and expected.**

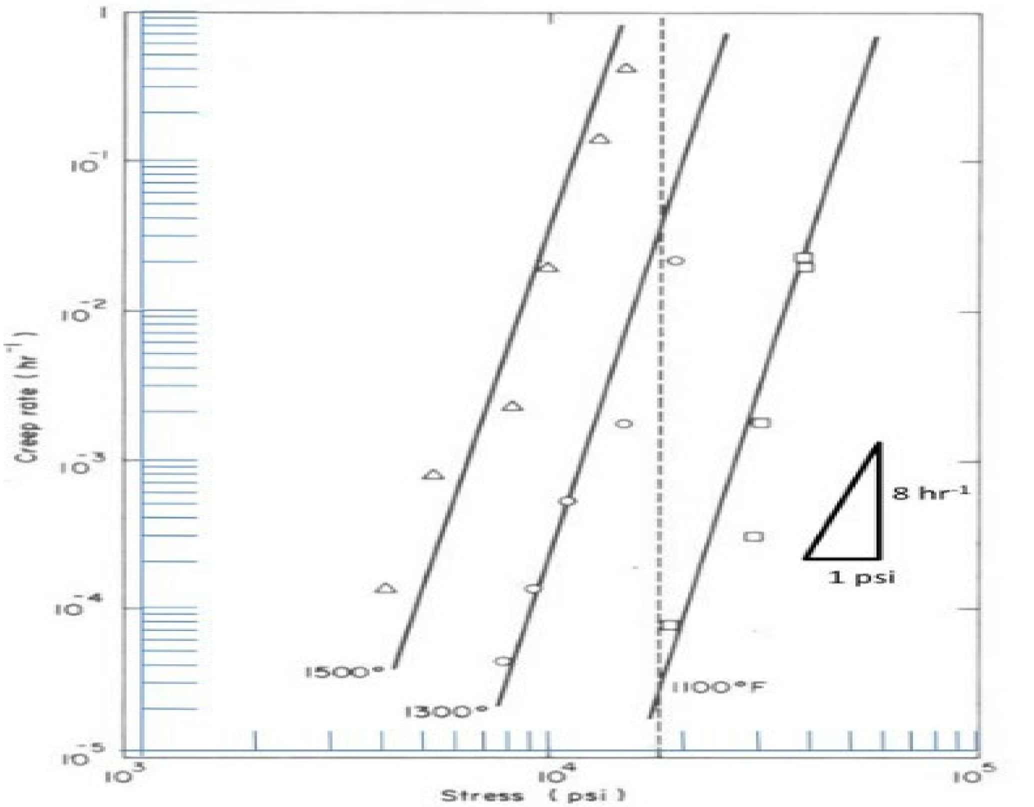
Boltzmann's Constant = 5.67×10^{-24} ft * lb/°Rankin **°Rankin=** °F + 460

Be clear about how you are handling the question of elastic, primary, and secondary creep strains. State all assumptions carefully.

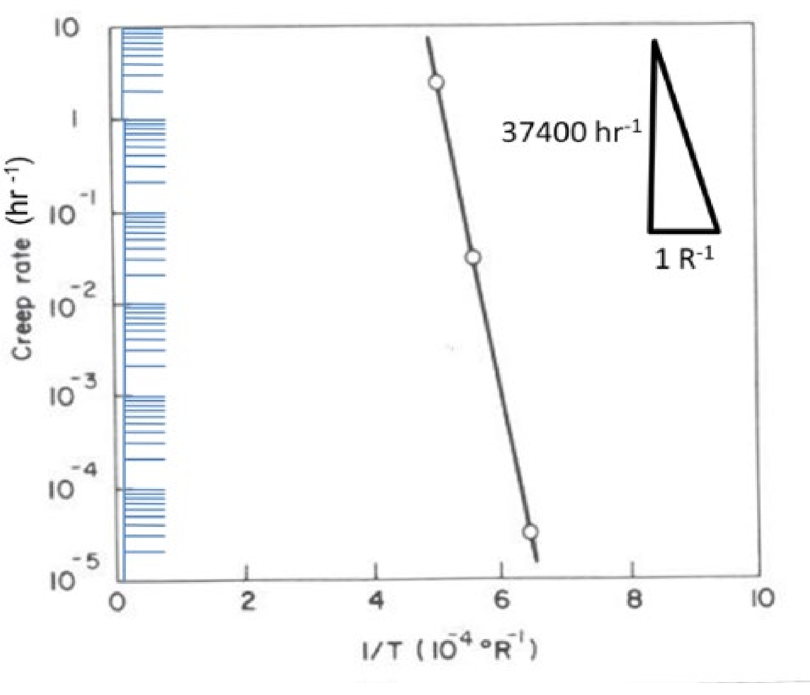
a) Determine m, H, and σ_0 from the experimental data (below on next page). To save you all some work, we have taken data from the first plot to generate the accompanying creep rate vs. 1/T curve. Slopes are as indicated. For simplicity, assume $\dot{\varepsilon}_0$ = 1 hr^{-1}

b) Another laboratory tests your material and finds the primary creep is responsible for an observed 0.76% strain and the material has elastic properties *E*= 29 Mpsi at room temperature and E = 22.3 Mpsi at 1100 F.

   You machine a cylindrical rod of diameter 0.25 in and load it in tension with a stress of 10,000 psi at 1100 F. What is the lifetime of the rod under these conditions if it is to be decommissioned upon reaching a total strain of 1.5%?



1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Creep rate (hr^{-1})
1500°
1300°
1100°F
8 hr^{-1}
1 psi
10^3
10^4
10^5
Stress (psi)



10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Creep rate (hr^{-1})
37400 hr^{-1}
1 R^{-1}
0
2
4
6
8
10
1/T (10^{-4} °R^{-1})

MSE 113 Summary of Essentials (Continuum Mechanics) R.O. Ritchie

(Elasticity/Plasticity)

Equilibrium (assuming no body forces or acceleration)

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} = 0$$
$$\frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{32}}{\partial x_3} = 0$$
$$\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} = 0$$

Compatibility

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Strains in Cylindrical Coordinate:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} \quad ; \quad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \quad ; \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$$\varepsilon_{zr} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \quad ; \quad \varepsilon_{r\theta} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial r} + \frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r}\right) \quad ; \quad \varepsilon_{z\theta} = \frac{1}{2}\left(\frac{\partial u_z}{r\partial \theta} + \frac{\partial u_\theta}{\partial z}\right)$$

Strains in Spherical Coordinates:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} \quad ; \quad \varepsilon_{\phi\phi} = \frac{1}{r}\frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r} \quad ; \quad \varepsilon_{\theta\theta} = \frac{1}{r\sin\phi}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} + \frac{u_\phi}{r}\cot\phi$$

$$\varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r\sin\phi}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right) \quad ; \quad \varepsilon_{r\phi} = \frac{1}{2}\left(\frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r} + \frac{1}{r}\frac{\partial u_r}{\partial \phi}\right)$$

$$\varepsilon_{\phi\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \phi} - \frac{u_\theta}{r}\cos\phi + \frac{1}{r\sin\phi}\frac{\partial u_\phi}{\partial \theta}\right)$$

Definitions

Hydrostatic stress:

$$\sigma = \frac{1}{3}\sum_{i=1}^{3}\sigma_{ii}$$

Dialation:

$$\varepsilon = \sum_{i=1}^{3}\varepsilon_{ii} \approx \frac{\Delta V}{V}$$

Deviatoric Stress:

$$\sigma'_{ij} = \sigma_{ij} - \delta_{ij}\sigma \quad \begin{cases}\delta_{ij} = 1, & i = j \\ \delta_{ij} = 0, & i \neq j\end{cases}$$

Deviator Strain:

$$\varepsilon'_{ij} = \varepsilon_{ij} - \delta_{ij}\frac{\varepsilon}{3} \quad \begin{cases}\delta_{ij} = 1, & i = j \\ \delta_{ij} = 0, & i \neq j\end{cases}$$

$$\bar{\sigma} = \left\{\frac{1}{2}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2] + 3(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2)\right\}^{1/2}$$

$$\left(d\bar{\varepsilon}_p\right)^2 = \frac{4}{9}\left\{\frac{1}{2}\left[\left(d\varepsilon_{11}^p - d\varepsilon_{22}^p\right)^2 + \left(d\varepsilon_{22}^p - d\varepsilon_{33}^p\right)^2 + \left(d\varepsilon_{33}^p - d\varepsilon_{11}^p\right)^2\right] + 3\left(d\varepsilon_{12}^{p2} + d\varepsilon_{23}^{p2} + d\varepsilon_{13}^{p2}\right)\right\}$$

Constitutive Relations

Elasticity

$$\varepsilon_{11} = \frac{1}{E}[\sigma_{11} - v(\sigma_{22} + \sigma_{33})]\,; \quad \varepsilon_{12} = \frac{\sigma_{12}}{2G}$$

$$\varepsilon_{22} = \frac{1}{E}[\sigma_{22} - v(\sigma_{33} + \sigma_{11})]\,; \quad \varepsilon_{23} = \frac{\sigma_{23}}{2G}$$

$$\varepsilon_{33} = \frac{1}{E}[\sigma_{33} - v(\sigma_{22} + \sigma_{11})]\,; \quad \varepsilon_{13} = \frac{\sigma_{13}}{2G}$$

$$G = \frac{E}{2(1 + v)} \qquad B = \frac{E}{3(1 - 2v)} = \frac{\sigma}{\varepsilon}$$

Plasticity

$$d\varepsilon_{11}^p = \frac{d\bar{\varepsilon}_p}{\bar{\sigma}}\left[\sigma_{11} - \frac{1}{2}(\sigma_{22} + \sigma_{33})\right]\,; \quad d\varepsilon_{12}^p = \frac{3}{2}\frac{d\bar{\varepsilon}_p}{\bar{\sigma}}\sigma_{12}$$

$$d\varepsilon_{22}^p = \frac{d\bar{\varepsilon}_p}{\bar{\sigma}}\left[\sigma_{22} - \frac{1}{2}(\sigma_{33} + \sigma_{11})\right]\,; \quad d\varepsilon_{23}^p = \frac{3}{2}\frac{d\bar{\varepsilon}_p}{\bar{\sigma}}\sigma_{23}$$

$$d\varepsilon_{33}^p = \frac{d\bar{\varepsilon}_p}{\bar{\sigma}}\left[\sigma_{33} - \frac{1}{2}(\sigma_{22} + \sigma_{11})\right]\,; \quad d\varepsilon_{13}^p = \frac{3}{2}\frac{d\bar{\varepsilon}_p}{\bar{\sigma}}\sigma_{13}$$

$$i.e., \qquad d\varepsilon_{ij}^p = \frac{3}{2}\frac{d\bar{\varepsilon}_p}{\bar{\sigma}}\sigma_{ij}$$

Yield Criteria

$$\tau_{max} = k\,; \ \bar{\sigma} = Y$$