1. (10 pts.) True/False and fill in the blank
<u>CIRCLE</u> T or F indicating whether the statement is true or false

a). The effective electron mass is inversely proportional to $\frac{\partial^2 E}{\partial k^2}$. T F

b). Metals have higher conductivities than semiconductors primarily because they have higher mobilities. more free carriers! T F

c). Metals have band gaps immediately above E_F. E_F is inside a band for metals! T F

d). The conductivity of both metals and intrinsic semiconductors decreases with increasing temperature. only for metals! T F

e). What is the band gap of Ge? 0.67 eV

f). The bonding in GaAs is <u>completely</u> covalent in nature. T F

g). Holes do not obey Fermi-Dirac statistics, but electrons do. T F

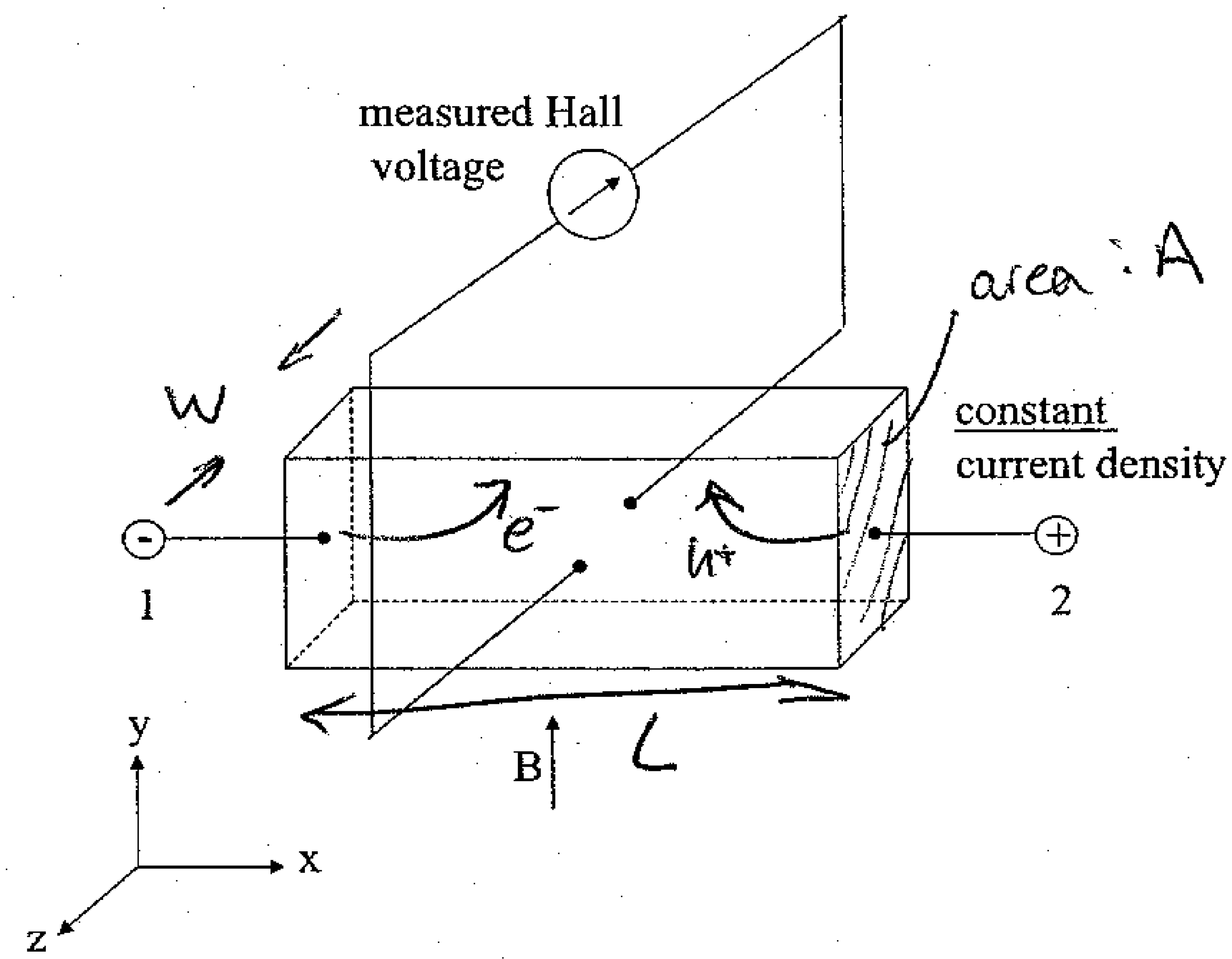
h). An electron has a larger DeBroglie wavelength than a proton traveling at the same velocity. T F

i) Acoustic phonons have longer wavelengths than optical ones in a given material. $0 \leq k \leq \frac{\pi}{a}$, for both A & O phonons. T F

j). The real-space and k-space widths of a wavepacket are directly proportional. inversely T F

2. **Hall Effect (14 points)**

The diagram below shows an experimental Hall effect setup. The front side of the sample has a positive potential with respect to the back side of the sample.



a) (2 pts) In which direction will holes be deflected by the magnetic field?
In which direction will electrons be deflected by the magnetic field?

$F = q\,\vec{v} \times \vec{B}$ by right hand rule: both are deflected to backside of sample ($-z$ direction)

charge: + for h^+
− for e^-

b) (6 pts) How would the Hall voltage change (become larger, smaller, or stay the same) for an intrinsic semiconductor if we increase the temperature of the sample? Give a justification for your answer.

$E_H = \frac{JB}{eN_e}$

$V_H = \frac{JBW}{eN_e} \Rightarrow V_H \propto \frac{1}{N_e}$

increase temp. → increase N_e

$\boxed{V_H \downarrow \text{ as } T \uparrow}$

c) (6 pts) How would the Hall voltage and the voltage between 1 and 2 change if we suddenly increased the carrier mobility in the sample? Give a justification for your answer.

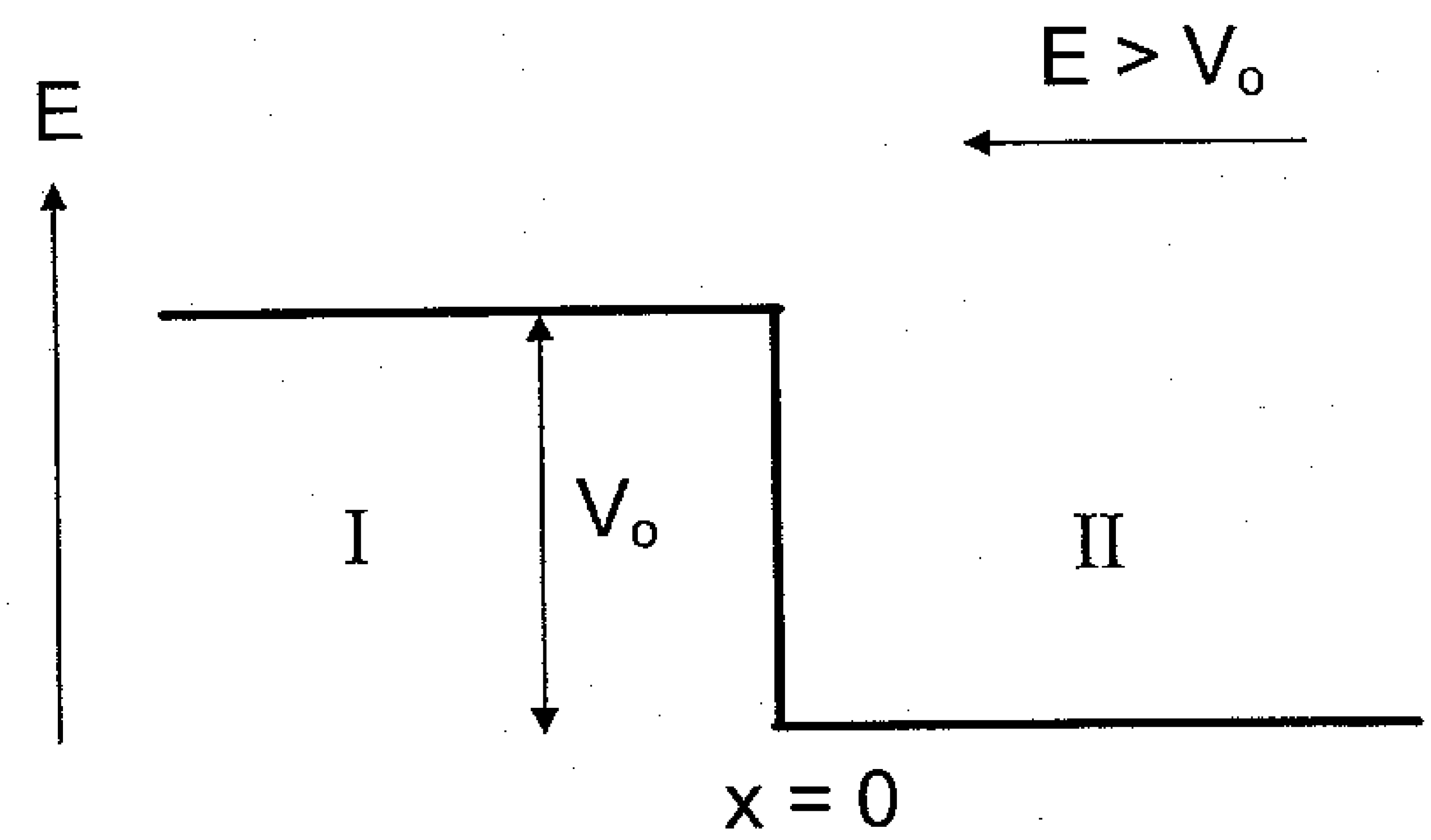
$V_H \neq f(\mu)$ → since the current density is constant, the Hall voltage does not change with increased mobility

$V_{12} = \frac{JL}{e\mu N}$

$\boxed{\mu \uparrow \text{ gives } V_{12} \downarrow}$

$V = IR$, $R = \frac{\rho L}{A}$, $\rho = \frac{1}{\sigma} = \frac{1}{e\mu N}$

3. Potential Barrier (12 pts)



Consider an electron of mass m incident from the right on the potential step shown above with an energy E greater than the magnitude (V_o) of the step.

a) (8 pts) Write down the solutions to the Schrödinger equation in the two regions (I&II). Define any variables you introduce in terms of the quantities given above. Which coefficients can we set to zero on physical grounds? Write down the boundary conditions that may be applied to solve this problem (you do *not* need to solve).

$$\psi_1(x) = \overset{0}{\cancel{A}}\exp(ik_1 x) + \overset{\text{transmitted}}{B\exp(-ik_1 x)}$$

$$\psi_2(x) = \underbrace{C\exp(ik_2 x)}_{\text{reflected}} + \underbrace{D\exp(-ik_2 x)}_{\text{incident}}$$

$$k_1 = \left(\frac{2m[E-V_o]}{\hbar^2}\right)^{1/2} \qquad k_2 = \left(\frac{2mE}{\hbar^2}\right)^{1/2}$$

→ "A" can be set equal to zero since there is nothing to reflect the e^- once it enters region I.

→ B.C.: $\psi(x)$ continuous, $\partial\psi/\partial x$ → continuous @ $x=0$

b) (4 pts) Describe qualitatively what would happen if an electron was incident from the right with energy less than the barrier height ($E < V_o$).

The e^- would tunnel into the barrier with a decaying exponential wavefunction: $\psi_1(x) = B'\exp(-k_1' x)$

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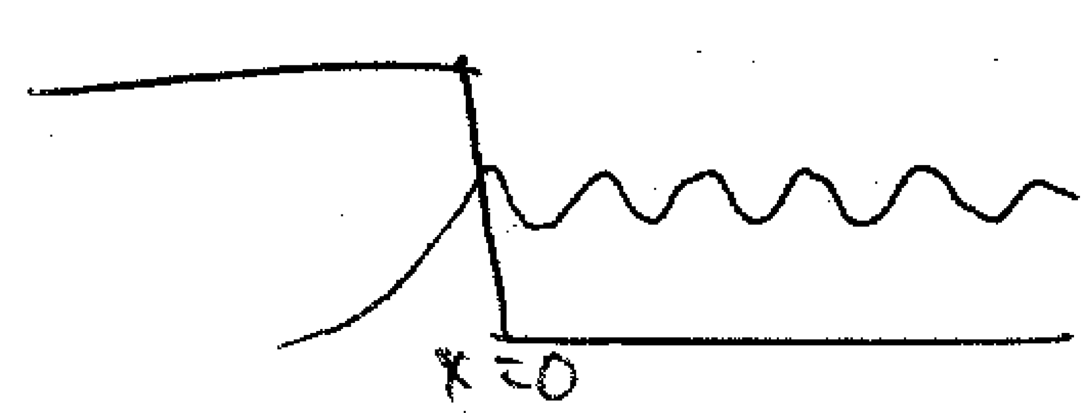
In region II, the solution is oscillatory

2

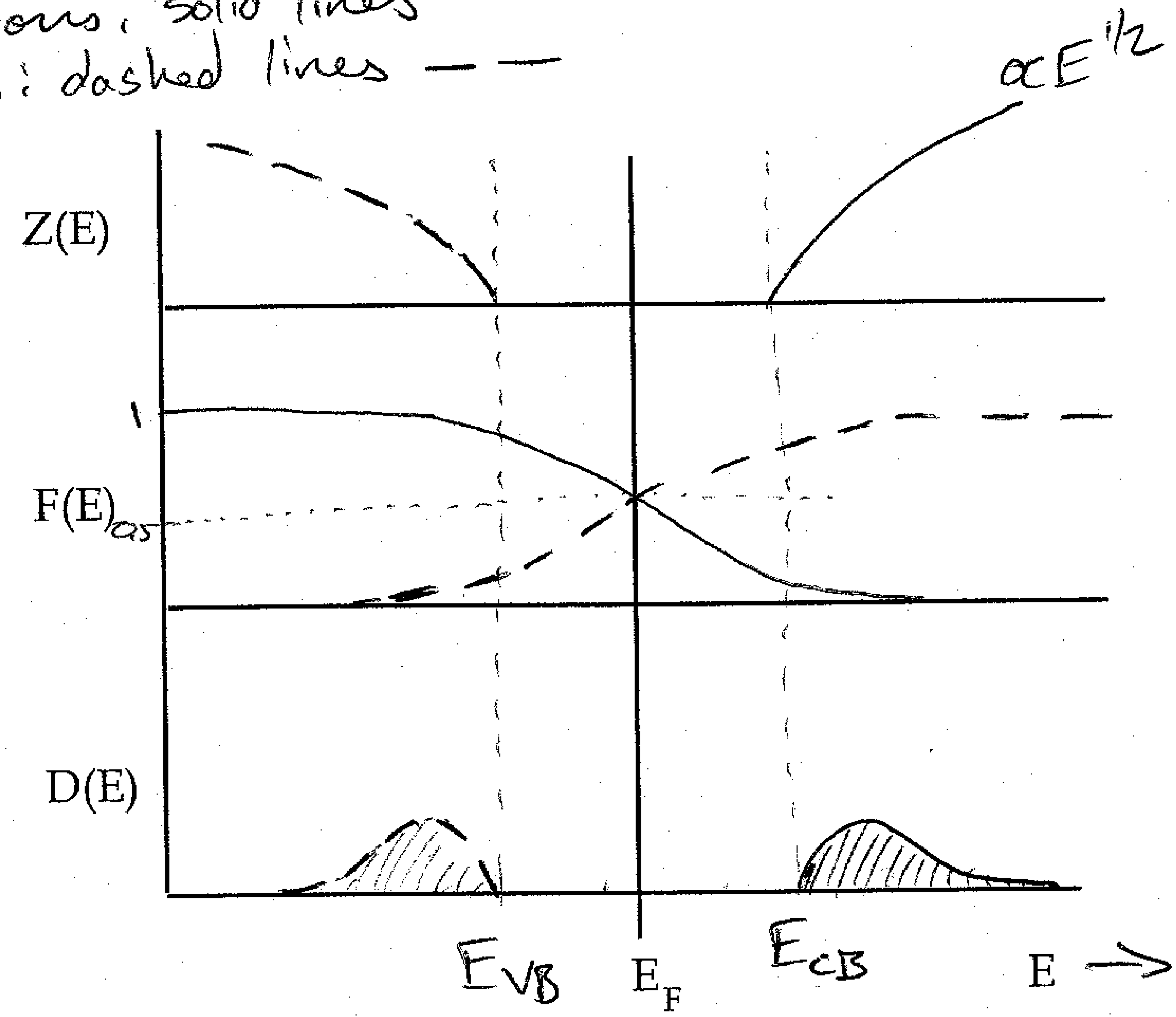


4. **(23 pts.) Semiconductor Statistics**

a) (9pts.) Sketch Z(E) vs. E, F(E) vs. E, and D(E) vs E with a common E axis for an intrinsic semiconductor at T > 0 K near the band edge. Clearly indicate the positions of the valence band edge and the conduction band edge. (Hint: Z(E): density of states, F(E): Fermi-Dirac function, D(E): density of occupied states.)

electrons: solid lines —

holes: dashed lines – –



b) (2 pts.) At what temperature is there a 50% probability that a state with an energy level at the Fermi energy will be occupied by an electron?

all temperatures

c) (6 pts.) At what temperature is there a 2 percent probability that a state with an energy 0.2 eV below the Fermi energy will be occupied by a hole?

Same temp that there is a 2% probability of finding an e^- 0.2 eV above the Fermi energy:

$$0.02 = \exp\left(-\frac{0.2\,eV}{k_B T}\right) \quad \text{assuming } E - E_F \gg k_B T$$

$$\boxed{T = 593\,K}$$

d) (6 pts) In intrinsic semiconductors the concentration of free carriers is a strong function of temperature. The intrinsic carrier concentration in Si ($E_g = 1.1$ eV) at 300 K (room temperature) is 2×10^{10} cm^{-3}. Ignore the temperature dependencies of E_g, N_C, and N_V. At what temperature will the concentration be two orders of magnitude lower?

$$n_i = \underbrace{\sqrt{N_c N_v}}_{\text{constant}} \exp\left(-E_G/2k_BT\right)$$

$$2\times10^{10}\,cm^{-3} = \sqrt{N_c N_v}\exp\left(\frac{-1.1\,eV}{2k_B(300K)}\right)$$

$$\sqrt{N_c N_v} = 3.45\times10^{19}\,cm^{-3}$$

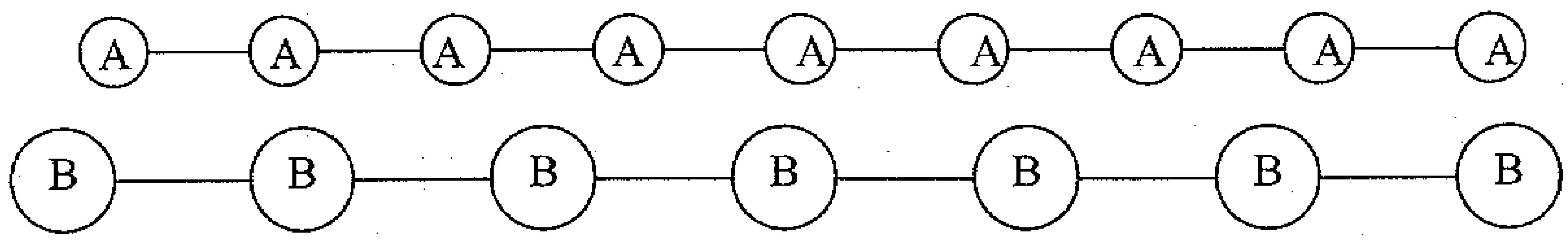
at what temp is $n_i = \dfrac{2\times10^{10}\,cm^{-3}}{100} = 2\times10^{8}\,cm^{-3}$

$$2\times10^{8}\,cm^{-3} = 3.45\times10^{19}\,cm^{-3}\exp\left(-\frac{1.1\,eV}{2k_B T}\right)$$

$$\boxed{T = 247\,K}$$

5. (21 pts.) Bonding and phonons

Consider two 1-D monatomic chains of two different atoms A and B:



The properties of the two chains are as listed:

Chain	Atom mass	Bonding potential as a function of bond length
A	m	$\phi(r) = 3\varepsilon[(\frac{a}{r})^{12} - 2(\frac{a}{r})^{6}]$
B	$3m$	$\phi(r) = 4\varepsilon[(\frac{2a}{r})^{12} - 2(\frac{2a}{r})^{6}]$

a) (4 pts.) Find the equilibrium bond length for both A and B chains.

$$\boxed{\left.\frac{d\phi(r)}{dr}\right|_{r=r_0} = 0}$$

$$A: \left.\frac{d\phi(r)}{dr}\right|_{r=r_0} = 3\varepsilon a^6\left[\frac{-12a^6}{r_0^{13}} + \frac{12}{r_0^{7}}\right] = 0$$

$$\Rightarrow \boxed{r_{0(A)} = a}$$

$$B: \left.\frac{d\phi(r)}{dr}\right|_{r=r_0} = 4\varepsilon(2a)^6\left[-\frac{12(2a)^6}{r^{13}} + \frac{12}{r^7}\right] = 0 \Rightarrow \boxed{r_{0(B)} = 2a}$$

b) (5 pts.) The bonding force at the equilibrium bond length can be approximated as that of an ideal spring which obeys Hooke's law. What is the ratio of the spring constants for the two chains $\frac{C_A}{C_B}$?

$$\boxed{C \propto \left.\frac{d^2\phi(r)}{dr^2}\right|_{r=r_0}}$$

$$A: \left.\frac{d^2\phi(r)}{dr^2}\right|_{r=r_0} = 36\varepsilon a^6\left[\frac{13a^6}{r_0^{14}} - \frac{7}{r_0^{8}}\right]$$

Plug in $r_0 = a$ $= \frac{216\varepsilon}{a^2}$

$$B: \left.\frac{d^2\phi(r)}{dr^2}\right|_{r=r_0} = 48\varepsilon(2a)^6\left[\frac{13(2a)^6}{r_0^{14}} - \frac{7}{r_0^{8}}\right]$$

Plug in $r_0 = 2a$ $\frac{72\varepsilon}{a^2}$

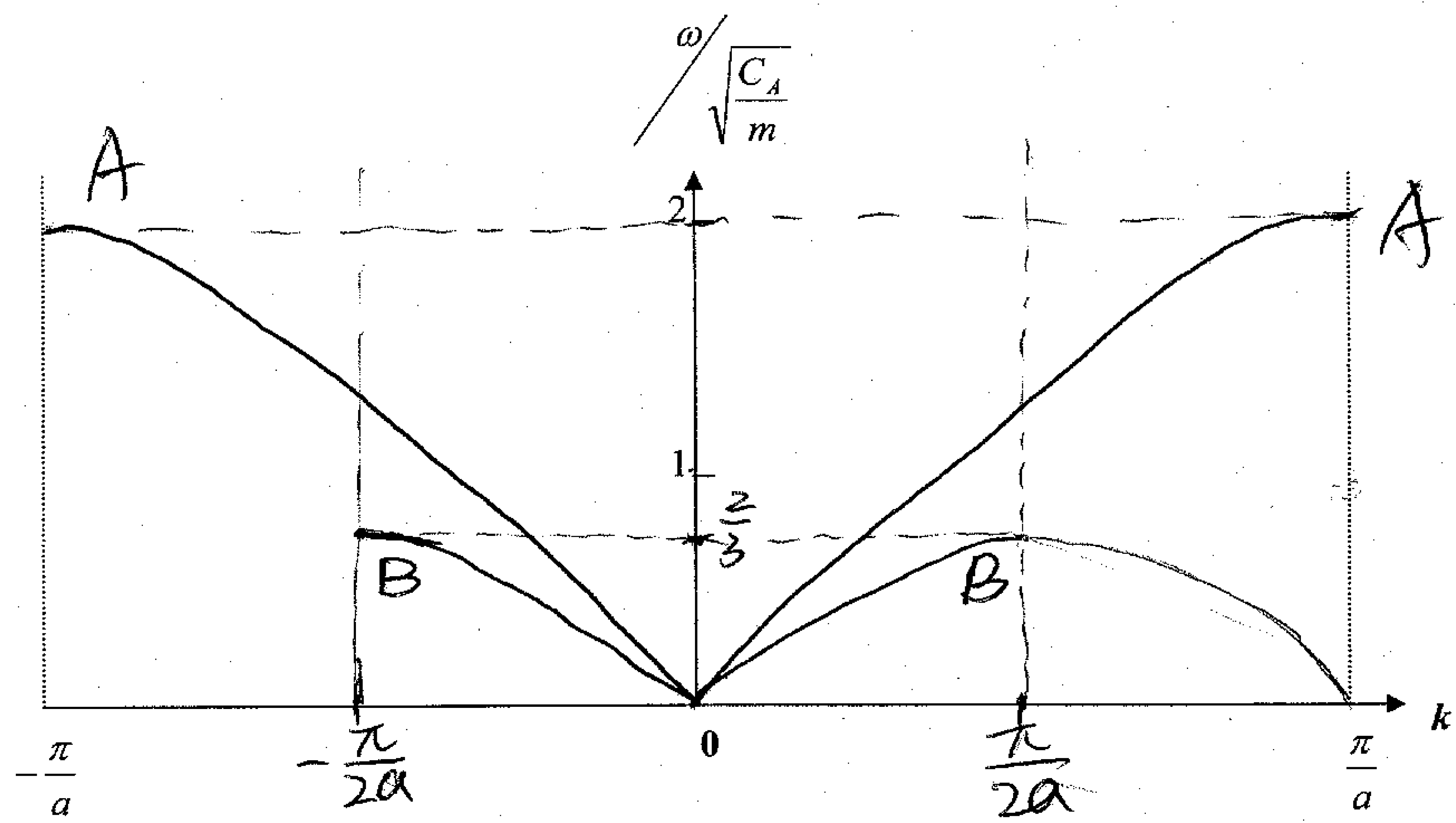
$$\therefore \frac{C_A}{C_B} = \frac{\frac{216\varepsilon}{a^2}}{\frac{72\varepsilon}{a^2}} = 3.$$

@ ZB:

$$\omega_A = 2\sqrt{\frac{C_A}{m}}, \quad \omega_B = 2\sqrt{\frac{C_B}{3m}} = \frac{2}{3}\sqrt{\frac{C_A}{m}}.$$

c) (6 pts.) Use results from a) and b) to draw phonon dispersion curves for both A and B in the same scale provided below. Label your curves.

You can assume $\frac{C_A}{C_B} = 3$ and $r_B = 2r_A = 2a$ if you could not answer parts a) or b).



d) (6 pts.) Label the four branches on the phonon dispersion curve (for a semiconductor) below. Estimate the maximum speed of sound in the semiconductor.

$K_{max}=\pi/a$, a = 5.64 Å

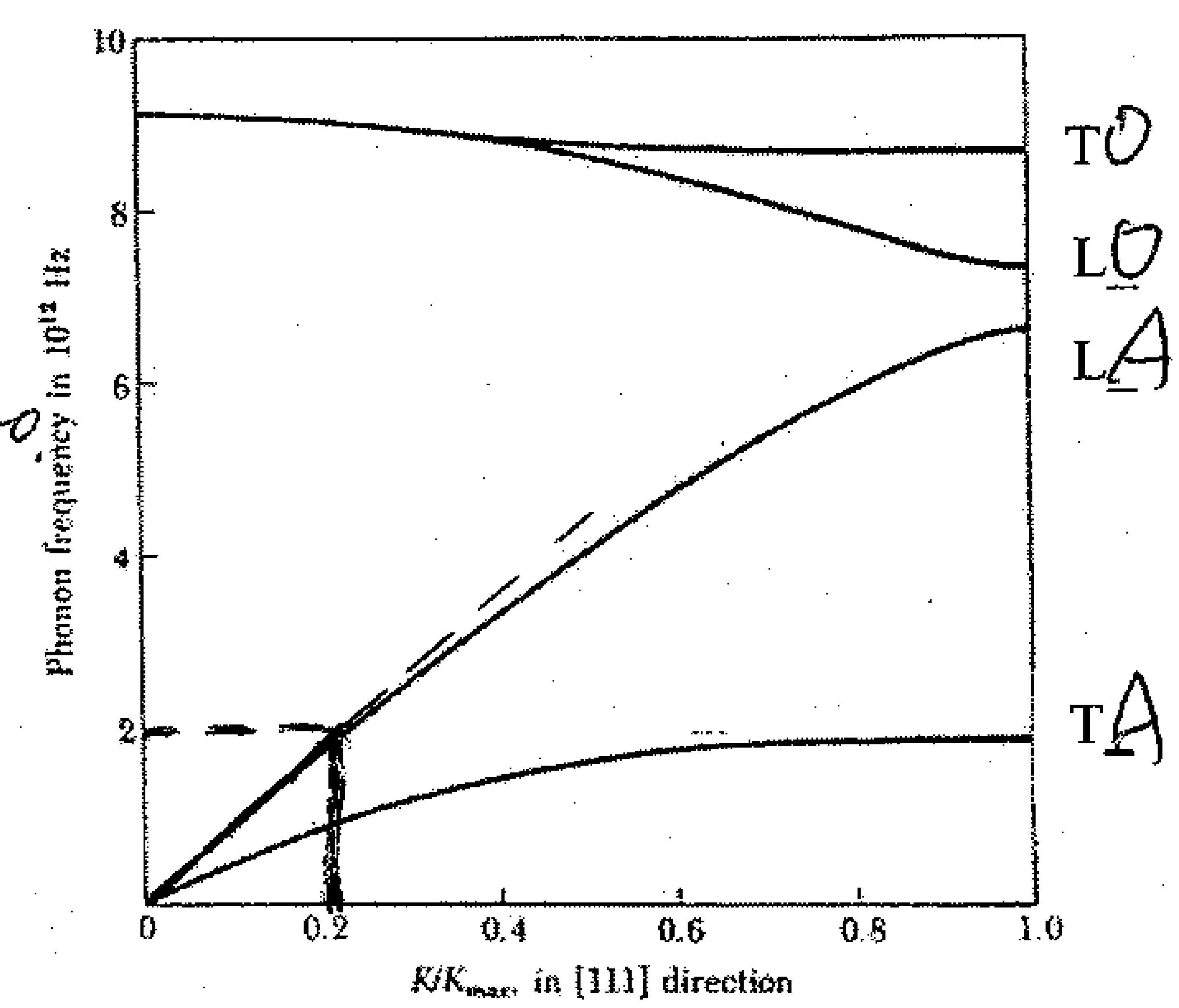
$$v_g = \frac{d\omega}{dk} = 2\pi \cdot \frac{df}{dk}.$$

$\therefore v_{sound} = 2\pi \cdot$ slope near $k=0$

$$\approx 2\pi \cdot \frac{2\times10^{12}\,[s^{-1}]}{0.2 \times \frac{\pi}{5.64\times10^{-10}\,[m]}}$$

$\approx 10^4$ m/s

[any value around 10^4 m/s is reasonable].



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