EECS 16A Designing Information Devices and Systems I
Spring 2019 Final Exam

Exam Location: Cory 144MA (DSP)

PRINT your student ID: ________________________________

PRINT AND SIGN your name: ________________, ________________ ________________
(last name) (first name) (signature)

PRINT the time of your Monday section and the GSI's name: ________________________________

PRINT the time of your Wednesday section and the GSI's name: ________________________________

Name and SID of the person to your left: ________________________________

Name and SID of the person to your right: ________________________________

Name and SID of the person in front of you: ________________________________

Name and SID of the person behind you: ________________________________

1. What was your favorite part of EE 16A? (1 point)

2. What are you looking forward to over the summer break? (1 point)

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

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3. Sensor Calibration (20 points)

You are setting up a lab experiment that involves measuring the position s of an object on a line. For this task, you are given a sensor that outputs a voltage based on the position of an object, as shown in Figure **??**. Your goal is to determine a function that returns the best approximation of the object's position based on the sensor's output voltage. You measure the output voltage of the sensor for some positions and get the data in Table **??**.

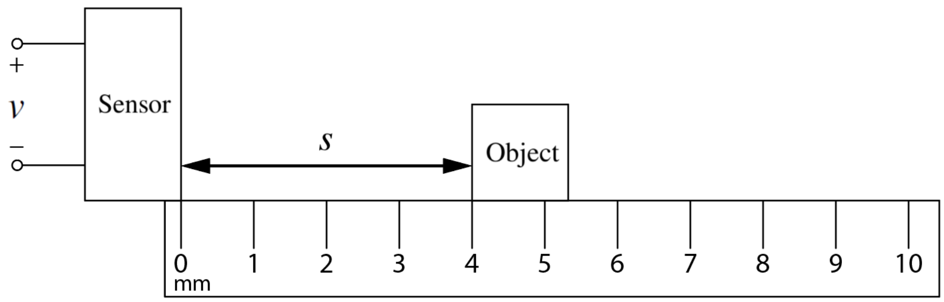

Figure 3.1: Experimental setup for sensor calibration.

Object's Position s (mm)	1	2	3
Sensor's Output Voltage v (mV)	4	10	15

Table 3.1: Sensor measurements for part (a).

Recall that m is *milli-*, or 10^{-3}.

(a) (4 points) After plotting the data from Table **??**, you notice it is fairly linear. So, you first assume that position is approximated by scaled voltage:

$$s \approx kv$$

where k is a scalar constant. Notice that the points don't exactly lie on a line due to noise. **Find k that minimizes the squared error $\|\vec{s} - k\vec{v}\|^2$.**

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(b) (6 points) You measure the output voltage to be -3 at $s = 0$, so you realize there must be a bias on the sensor. Your updated data is in Table **??**.

Object's Position s (mm)	0	1	2	3
Sensor's Output Voltage v (mV)	-3	4	10	15

Table 3.2: Sensor measurements for part (b).

You revise your equation as follows:

$$s \approx kv + b$$

where b is a scalar constant representing the bias. **Set up the least squares problem to find k and b using data from Table ??. Derive an expression for k and b (i.e., $\begin{bmatrix} k \\ b \end{bmatrix} =???$), as a function of table data, but DO NOT SOLVE.**

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(c) (6 points) Using a different sensor, you take more data points and find that these points don't follow your model from (b) very well:

Object's Position s (mm)	4	5	6	7
Sensor's Output Voltage v (mV)	20	45	98	214

Table 3.3: Sensor measurements for part (c).

By inspection, you decide either a quadratic fit or a logarithmic fit may work. The equations for these mathematical models are as follows:

$$s_{quadratic} \approx a_2 v^2 + a_1 v + a_0$$
$$s_{logarithmic} \approx b_1 ln(v) + b_0$$

where a_0, a_1, a_2, b_0, and b_1 are constants. **Set up the least squares problems for the two models using data only from Table ??. Derive expressions for the constants (i.e.,** $\begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix}$ =??? **and** $\begin{bmatrix} b_1 \\ b_0 \end{bmatrix}$ = ???)**, as functions of table data, but DO NOT SOLVE.**

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(d) (4 points) After fitting the models from part (c), assume you calculate the following errors $e = s_{approximate} - s_{measured}$ for the sets of data in that part:

Object's Position s (mm)	4	5	6	7
$e_{quadratic}$ (mm)	1	-1.5	0.5	0
$e_{logarithmic}$ (mm)	-0.9	0	-1	1.1

In the context of least squares, which is the better fit? Justify your answer mathematically.

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4. I think *neuron* to something! (18 points)

Figure **??** shows a diagram of a neural cell membrane. There are pumps and channels for both sodium (Na^+) and potassium (K^+).

Despite their names, the channels act as current sources while the pumps behave like resistors. The membrane itself acts as a capacitor, as it is able to build up charge on either side.

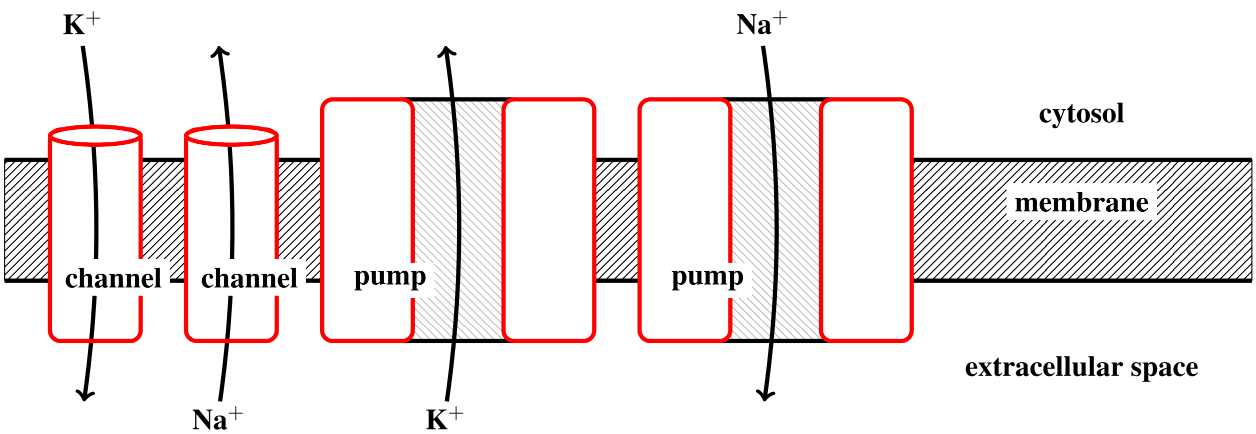

Figure 4.1: Diagram of the neural membrane

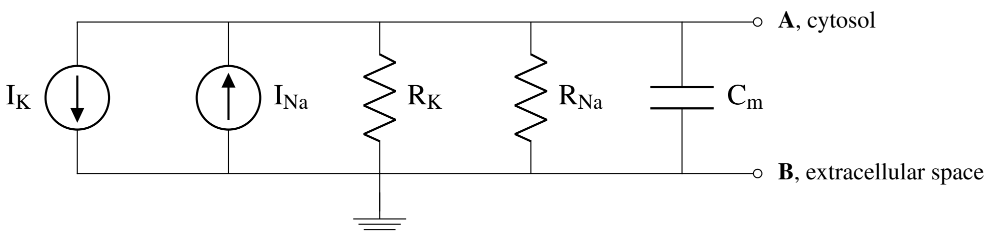

Figure 4.2: Circuit model of the neural membrane

(a) (3 points) **In Figure ??, explain why, if we attach different loads and measure the currents and voltages through the loads, we will not be able to find values for I_K and R_K.**

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(b) (4 points) Thankfully, some of your biologist friends know how to remove the sodium channel and pump so you can measure I_K and R_K. Doing this, you are left with the following:

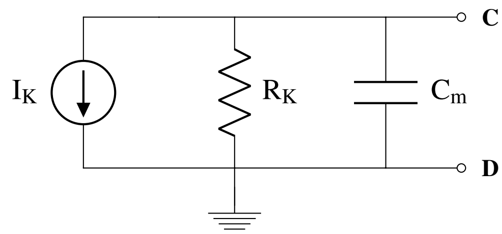

Figure 4.3: Membrane with sodium (Na^+) complexes removed

At steady state, you measure an open-circuit voltage of $V_{CD} = -100\,\text{mV}$. Also at steady state, you find a short-circuit current of $I_{CD} = -50\,\text{pA}$. **Find $\mathbf{I_K}$ and $\mathbf{R_K}$.**
Hint: recall that p (pico) is 10^{-12}.

(c) (4 points) Similarly, you find that $I_{Na} = \frac{1}{100} \cdot I_K$ and $R_{Na} = 100 \cdot R_K$. **Find V_{AB} in Figure ?? in terms of $\mathbf{I_K}$ and $\mathbf{R_K}$.**

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(d) (7 points) Due to the high values of the resistors and the solution to part (c), we will now neglect the resistors and passive sodium channels.

Neurons transmit signals from one to the other via neurotransmitters. When a nearby neuron releases the neurotransmitter acetylcholine, special acetylcholine-gated sodium channels open wide, allowing a large amount of sodium $I_{Na}^{(d)}$ to rush into the cytosol.

If $I_{Na}^{(d)}$ is on for long enough, the membrane voltage changes enough that it reaches a critical value V_{ref}, triggering a neural response.

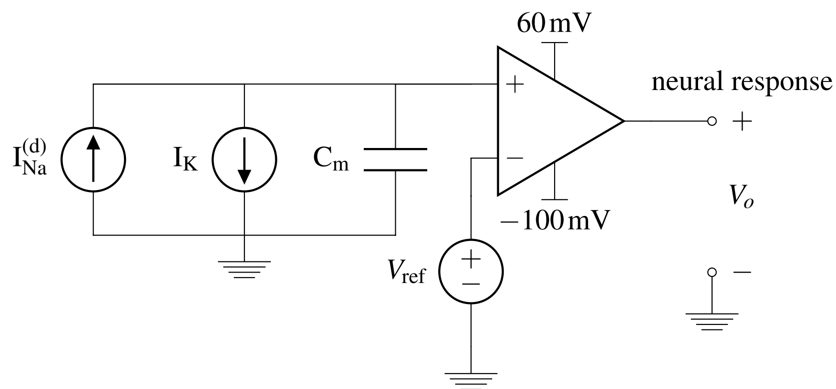

Figure 4.4: Triggerable membrane

Assuming:

- $I_K = 50\,pA$
- $I_{Na}^{(d)} = 150\,pA$
- $C_m = 20\,pF$
- $V_{ref} = -50\,mV$
- V_+ is initially $-100\,mV$,

How long does it take to trigger a response from the neuron?

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5. Tracking your TA Terry (10 points)

Terry is a very mischievous child, and his mother is interested in tracking him. Terry sends meetup locations in the form of a **two-dimensional real vector** to his friends. His mother is convinced he is encoding his messages in a different basis, but she doesn't quite know how, so she asks for your help in figuring it out.

For this problem, the $\mathbb{R}^2$ standard basis vectors will be denoted by

$$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(a) (2 points) You are given that the text sent by Terry is $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$. The mom only knows the **standard basis**, while Terry is using basis vectors $\vec{v}_1$ and $\vec{v}_2$. Based on this information, **write the location each person thinks the text refers to as a linear combination of the given basis vectors:** $\vec{e}_1, \vec{e}_2, \vec{v}_1, \vec{v}_2$**. Label your answer for the mother as** $\vec{r}_m$ **and** $\vec{r}_t$ **for Terry.**

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(b) (3 points) Now you're told that Terry sent a text $\begin{bmatrix}1\\4\end{bmatrix}$. Terry's friend tells you that Terry's location is given by the vector $\begin{bmatrix}8\\9\end{bmatrix}$. Assuming this vector represents the standard basis coordinates of Terry's actual location, **determine the unique basis vectors he is using, or if it is impossible–explain why.**

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(c) (5 points) As it turns out, Terry is a nice guy and is not doing anything mischievous after all. To prove it, he decides to help his mom learn how he's encoding his messages. He gives his mom a new set of basis vectors **P**:

$$\vec{p}_1 = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \text{ and } \vec{p}_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

and tells her his own basis vectors:

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

He challenges her to determine the coordinates (in the **P** basis), corresponding to the location $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ in his own basis. **Determine the coordinates, or if the task is impossible–explain why.**

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6. Automatic Bitcoin Investment Portfolio (18 points)

Your GSI Sang Min hears about the hype that surrounds Bitcoin and decides to invest some of his salary in Bitcoin. Sang Min is extremely lazy and wants to choose from one of two automatic payment systems that buy a set amount of money's worth of Bitcoin at the beginning of every quarter in the financial calendar year. The systems then sell the Bitcoin and return to Sang Min the money earned from selling that amount of Bitcoin at the end of every quarter. The following are the two available investment portfolio choices:

$$\text{Portfolio A}: \ \vec{p}_A = \begin{bmatrix}\$60 & \$40 & \$60 & \$40\end{bmatrix}^{\mathrm{T}}$$
$$\text{Portfolio B}: \ \vec{p}_B = \begin{bmatrix}\$20 & \$80 & \$20 & \$80\end{bmatrix}^{\mathrm{T}}.$$

If Sang Min selects Portfolio A, \$60 of Sang Min's money is invested in Bitcoin at the start of the first quarter, \$40 at the start of the second quarter, \$60 at the start of the third quarter, and \$40 at the start of the fourth quarter. Similarly, selecting Portfolio B invests \$20 in the first quarter, \$80 in the second quarter, and repeats.

The two portfolios above also offer Sang Min the choice of investing the first amount at the start of the second quarter, i.e. Sang Min can choose to invest \$60 in the first quarter, \$40 in the second quarter, and repeat **or** invest \$40 in the first quarter, \$60 in the second quarter, and repeat, if Portfolio A is selected.

The projected Bitcoin valuation at the end of a quarter with respect to its valuation at the beginning of the quarter for the next year in quarterly increment is represented by $\vec{x}$ below:

$$\vec{x} = \begin{bmatrix}+0.5 & -1 & +0.5 & -2\end{bmatrix}^{\mathrm{T}}.$$

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(a) (6 points) Recall that $\vec{x} = \begin{bmatrix} +0.5 & -1 & +0.5 & -2 \end{bmatrix}^T$. **Compute the circular cross-correlation of $\vec{x}$ and $\vec{p}_A$ with period $N = 4$**, i.e. find $\text{circcorr}(\vec{x}, \vec{p}_A)[k]$, **for $k = 0, 1, 2, 3$.** Similarly, **compute** $\text{circcorr}(\vec{x}, \vec{p}_B)[k]$ **for $k = 0, 1, 2, 3$.**

(b) (2 points) If Sang Min chooses Portfolio A, **which option ($60 first or $40 first)** should he take to maximize profit/minimize loss after one year? *Please bubble in your answer completely below and explain your answer.*

◯ $60 first ◯ $40 first

If Sang Min chooses Portfolio B, **which option ($20 first or $80 first)** should he take to maximize profit/minimize loss after a year? *Please bubble in your answer completely below and explain your answer.*

◯ $20 first ◯ $80 first

(c) (2 points) **Which portfolio and option** should Sang Min choose to maximize profit/minimize loss after a year? **How much money** did Sang Min make/lose over the year with this optimal choice? [*Hint: Sang Min invested $200 over the one year period.*]

For the remaining parts of this problem, consider the scenario where Sang Min chooses Portfolio A.

(d) (4 points) Write the circular cross-correlation vector of $\vec{x}$ and $\vec{p}_A$ as a matrix-vector product, i.e. write down the matrix $\mathbf{C}$ in the equation

$$\overrightarrow{\text{circcorr}}(\vec{x}, \vec{p}_A) = \mathbf{C}\vec{x}.$$

During one meeting with Professor Liu, Sang Min learns that Professor Liu made thousands of dollars in 2016 using Portfolio A. Sang Min is curious about the quarterly Bitcoin valuations during that time period that made Professor Liu rich, but Professor Liu does not remember what the valuations vector $\vec{y}$ was during that time period. Knowing only $\overrightarrow{\text{circcorr}}(\vec{y}, \vec{p}_A)$, we will investigate a method to recover $\vec{y}$.

(e) (4 points) Professor Liu recognizes that the matrix $\mathbf{C}$ from part (d) is a **circulant matrix** and gives Sang Min a matrix called $\mathbf{F}$ that diagonalizes $\mathbf{C}$, i.e.

$$\mathbf{C} = \mathbf{F}^{\mathrm{T}} \Lambda \mathbf{F},$$

where Λ is a diagonal matrix. $\mathbf{F}$ has a very special property: it is an orthogonal matrix that satisfies $\mathbf{F}^{-1} = \mathbf{F}^{\mathrm{T}}$, i.e. the inverse of the matrix is equal to the transpose of the matrix. Sang Min does not remember how to invert $\mathbf{C}$, but he knows how to invert a diagonal matrix. **What is $\vec{y}$ in terms of $\overrightarrow{\text{circcorr}}(\vec{y}, \vec{p}_A)$, $\mathbf{F}$, and Λ?**

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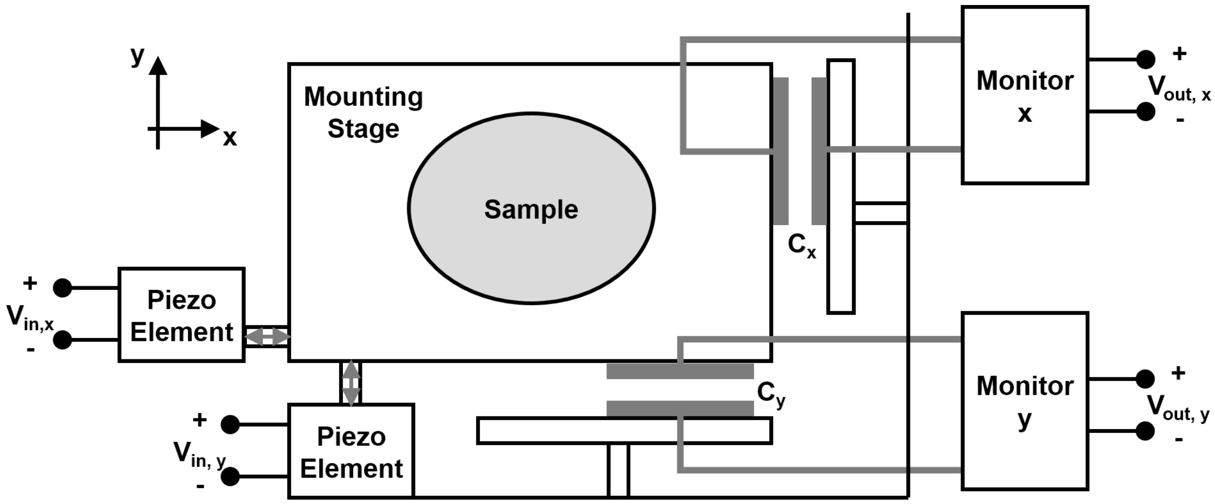
7. 2D Piezo positioner (20 points)

We would like to modify the Piezo positioner, the device we covered in Midterm 2, for use in two dimensions. The overall structure of a 2D Piezo positioner is shown below.



The mounting stage moves within the $x-y$ plane. There are two input voltages that control the location of the mounting stage along the x- and y- axes, respectively, as well as two capacitors. Each capacitance depends on the distance between the mounting stage and the measurement stage (in x and y, respectively).

Assume that there's a simple relationship between the input voltages and the outputs from the monitoring circuit, which is given as follows.

$$x_{\text{stage}} = \alpha_x V_{\text{in},x}, \quad V_{\text{out},x} = x_{\text{stage}} - x_{\text{desired}},$$
$$y_{\text{stage}} = \alpha_y V_{\text{in},y}, \quad V_{\text{out},y} = y_{\text{stage}} - y_{\text{desired}}.$$

We use the output voltages at time step i to update the input voltages at time step $i+1$ as shown by the equations below:

$$V_{\text{in},x}[i+1] = V_{\text{in},x}[i] - kV_{\text{out},x}[i],$$
$$V_{\text{in},y}[i+1] = V_{\text{in},y}[i] - kV_{\text{out},y}[i].$$

Let's say that we want to place the mounting stage at the origin ($x_{\text{desired}} = y_{\text{desired}} = 0$), and define the following:

$$\vec{V}_{\text{in}}[i] = \begin{bmatrix} V_{\text{in},x}[i] \\ V_{\text{in},y}[i] \end{bmatrix}$$
$$\vec{V}_{\text{out}}[i] = \begin{bmatrix} V_{\text{out},x}[i] \\ V_{\text{out},y}[i] \end{bmatrix}.$$

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(a) (5 points) We would like to express the relationship between $\vec{V}_{\text{in}}[i]$ and $\vec{V}_{\text{in}}[i+1]$. **Find A**, where

$$\vec{V}_{\text{in}}[i+1] = \mathbf{A}\vec{V}_{\text{in}}[i].$$

(b) (2 points) The initial input voltages are given as a two-dimensional vector $\vec{V}_{\text{in}}[0] = \begin{bmatrix} V_{\text{in},x}[0] \\ V_{\text{in},y}[0] \end{bmatrix}$, whose entries are not zero. **Express $\vec{V}_{\textbf{in}}[i]$ in terms of k, α_x, α_y, i and $\vec{V}_{\textbf{in}}[0]$.**

(c) (4 points) Eventually, we want to ensure $x_{\text{stage}} = x_{\text{desired}} = 0$ and $y_{\text{stage}} = y_{\text{desired}} = 0$ as $i \to \infty$. **What conditions do α_x and α_y have to satisfy to guarantee this?** Leave your answers in terms of α_x, α_y, and k.

(d) (2 points) It turns out the previously known relationship between the input voltages and the outputs are incorrect, and the following is the updated relationship.

$$x_{\text{stage}} = \alpha_{11} V_{\text{in},x} + \alpha_{12} V_{\text{in},y},\ V_{\text{out},x} = x_{\text{stage}} - x_{\text{desired}},$$
$$y_{\text{stage}} = \alpha_{21} V_{\text{in},x} + \alpha_{22} V_{\text{in},y},\ V_{\text{out},y} = y_{\text{stage}} - y_{\text{desired}}.$$

Find the matrix B such that $\vec{V}_{\text{in}}[i+1] = \mathbf{B}\vec{V}_{\text{in}}[i]$.

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(e) (7 points) Suppose that $k = 0.1$, $\alpha_{11} = 4$, $\alpha_{12} = 1$, $\alpha_{21} = -2$, and $\alpha_{22} = 7$. **Does this system converge to** $x_{\text{stage}} = y_{\text{stage}} = x_{\text{desired}} = y_{\text{desired}} = 0$? (*Hint: use diagonalization.*)

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8. Word Embeddings (12 points)

In Natural Language Processing, we often represent words by vectors that correspond to their meanings. These vectors are called word embeddings, and they have some nice properties. One property is that if we have the vector $\vec{x}_{\text{dog}}$ representing the word "dog" and the vector $\vec{x}_{\text{house}}$ representing the word "house," then the vector for "doghouse" is some linear combination

$$\vec{x}_{\text{doghouse}} = \alpha \vec{x}_{\text{dog}} + \beta \vec{x}_{\text{house}}.$$

We will call words like "dog" and "house" **simple words**, while words like "doghouse" are **compound words**. The embedding for a compound word can always be expressed as the linear combination of two simple word embeddings.

(a) (4 points) My friend has a matrix $\mathbf{A}$ where each column $\vec{a}_i$ is a simple word embedding and nonzero:

$$\mathbf{A} = \begin{bmatrix} | & | & & | \\ \vec{a}_1 & \vec{a}_2 & \dots & \vec{a}_n \\ | & | & & | \end{bmatrix}.$$

Suppose $\vec{x} = \alpha \vec{a}_u + \beta \vec{a}_v$ is a compound word embedding composed of two words from $\mathbf{A}$, where $\alpha, \beta \neq 0$. Also, suppose that the columns of $\mathbf{A}$ are orthogonal, so for $i \neq j$, $\vec{a}_i^{\mathrm{T}} \vec{a}_j = 0$.

Then, for which indices i will the inner product $\vec{x}^{\mathrm{T}} \vec{a}_i$ be non-zero? Justify your answer.

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(b) (4 points) Let

$$\mathbf{B} = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 1 \\ 1 & -1 & 3 \\ -2 & 2 & 3 \\ 6 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

be a matrix whose columns are simple word embeddings. Note that the columns are orthogonal. **Which two columns make up the following compound word embedding?**

$$\begin{bmatrix} 5 \\ 1 \\ 5 \\ -1 \\ 12 \\ -1 \end{bmatrix},$$

Justify your answer.

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(c) (4 points) Suppose $\mathbf{C}$ is an $n \times n$ matrix with columns $\vec{c}_1, \ldots, \vec{c}_n$, where we have

$$\vec{c}_i^{\mathrm{T}} \vec{c}_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}.$$

Prove that the inverse of C is $\mathbf{C}^{\mathrm{T}}$.

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9. Pineapple Pioneers: Part 1 (20 points)

Your TAs Nick and Linda are big fans of pineapple on their pizza. Unfortunately, they don't have enough money to buy a nice pineapple pizza, so they decide to make one instead. They decide to grow a pineapple plant in the dirt outside of Cory Hall, but the harsh environment makes it hard for the plant to grow! Bugs keep trying to eat their plant, so they decide to make an alarm system that will sound a horn and scare them away, but they need your help.

Note: m (milli) is 10^{-3}, *μ (micro) is* 10^{-6} *and n (nano) is* 10^{-9}.

(a) (6 points) You tell them a capacitive sensor might work. With your immense 16A knowledge, you construct a special capacitor C_{special} that will change capacitance values when a bug touches it, and place them all around the growing plant. The behavior of C_{special} is such that:

$$C_{\text{special}} = \begin{cases} C_1 & \text{Bug is present} \\ C_2 & \text{No bug is present} \end{cases}.$$

You decide to drive this capacitor with a current source of value $I_S = 1\mu A$ as shown below.

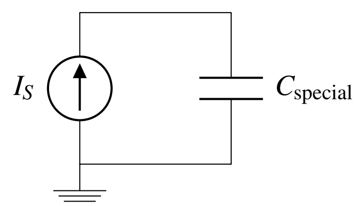

Figure 9.1: Capacitor charging circuit

You charge the capacitor for 1 second and record the voltage across it. You then discharge the capacitor and place a bug on it. You again allow it to charge for 1 second. The following behavior is observed:

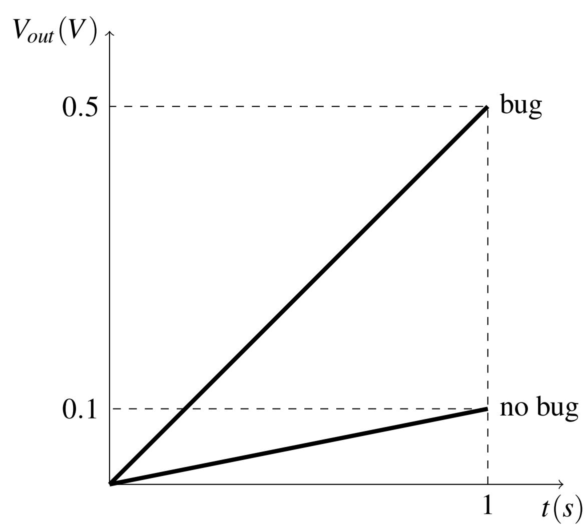

Figure 9.2: Capacitor behavior

Assume the capacitor is intially uncharged, and is fully discharged in between the two tests. Find values for C_1 and C_2 that give the behavior in Figure ??.

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(b) (8 points) Regardless of your answer to part (a), assume $C_1 = 5\text{nF}$ and $C_2 = 12\text{nF}$. Nick and Linda want to only check the voltage after some time, and output either 2V or 0V depending on if a bug is present or not to make things easier. You find that you have one op-amp, one voltage source of value 0.2V, and one voltage source of 2V.

Using the circuit from part (a) and the provided components only, design a circuit that after some time t, outputs 2V if a bug is present, and 0V if not. Also, determine the value of t after which the output is valid. Assume the capacitor is initially uncharged and that we are only checking the circuit once (i.e., don't worry about resetting the capacitor's charge). You must label the supplies on the op amp.

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(c) (5 points) While testing the circuit, your current source breaks! You manage to find another one, but it varies with time. The new circuit and behavior are shown below.

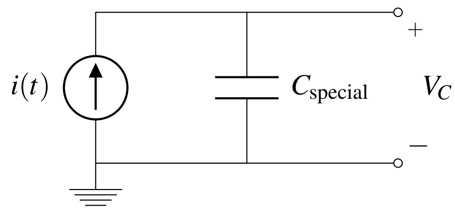
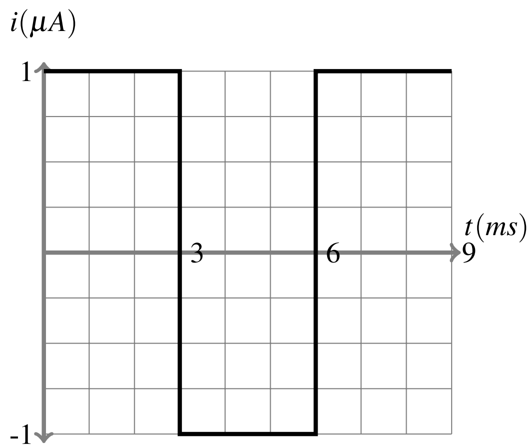
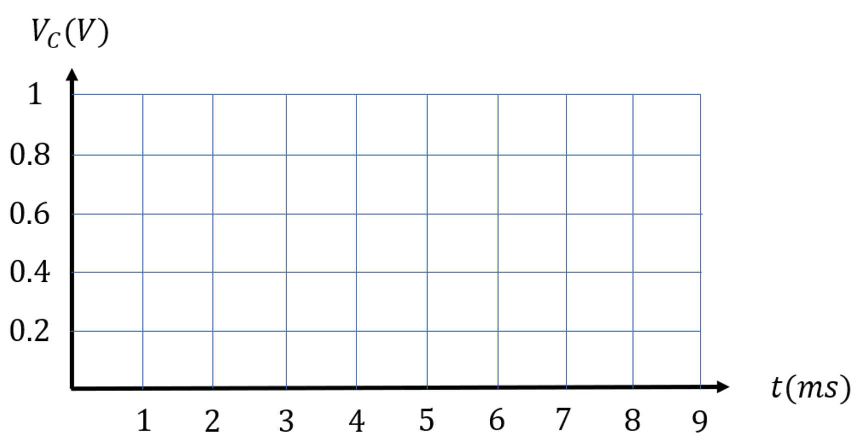


Figure 9.3: New current source



On the graph below, plot the voltage across the capacitor (V_C) as a function of time for the cases where a bug is present and where no bug is present. Clearly label which is which. Assume $C_1 = 5$nF and $C_2 = 12$nF. Assume the capacitor is initially uncharged.



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Optional room for work. NOT GRADED

(d) (1 point) Does pineapple belong on pizza? *You can put anything you want or nothing at all and you will get the points for this part.*

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10. Pineapple Pioneers: Part 2 (30 points)

Earlier in the exam, Nick and Linda tried to grow a pineapple plant outside Cory Hall, but found the environment to be too difficult to grow anything in. To increase their yield, they decide to ask Ryan for suggestions. He suggests that they move their operation to South America where the environment is better. Since it's so far away, Nick and Linda decide to monitor the plant wirelessly by sending data from various sensors with an antenna.

Back in Berkeley, with the help of Ryan, they come up with the following setup to retrieve the information and process it. The first device is an antenna, which you don't need to understand for this problem.

Note: m (milli) is 10^{-3} *and n (nano) is* 10^{-9}.

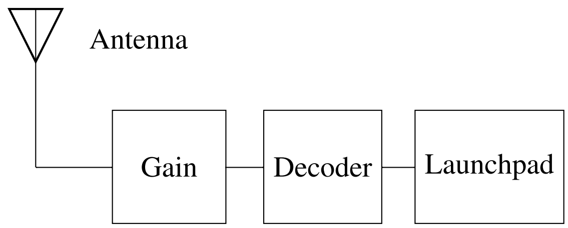

Figure 10.1: Receiver block diagram

Unfortunately, the best the trio can come up with is a block diagram, and they need your help implementing these blocks.

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(a) (4 points) Let's first start by looking at the antenna. The Thevenin equivalent of the antenna is shown in Figure **??**.

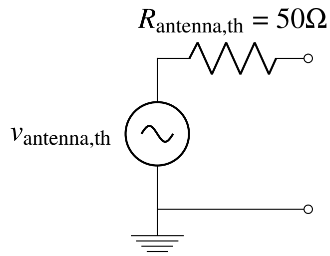

Figure 10.2: Antenna Thevenin equivalent

The resistance of the load attached to the antenna is very important for many reasons. One reason is to maximize power transfer. **If we were to attach a load resistor R_L to the open port on this circuit, what should the resistance be to have maximum power dissipated in R_L? Justify your answer.**

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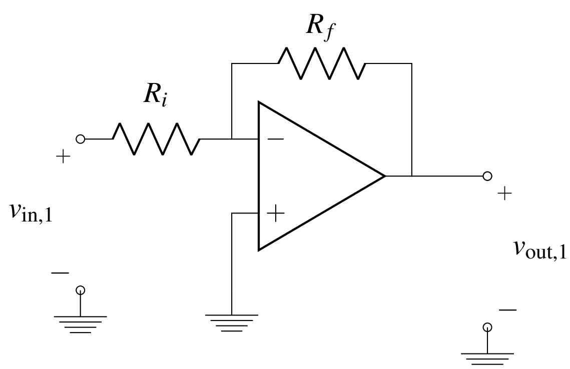
(b) (4 points) Let's now look at the "Gain" block. The signal will be significantly weakened when traveling through the air, so we need to apply gain to what comes out of the antenna or our Launchpad won't be able to tell what any of the data means. Ryan, Nick, and Linda ask their local wireless experts for a suggestion, and they suggest the following circuit:



They claim that you can make this circuit "look like" whatever resistance you want. To test their theory, **find the Thevenin resistance with respect to the port $v_{in,1}$.**

Hint: use the V_{test}/I_{test} method.

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(c) (4 points) We now want to finish this circuit. Suppose that we need an overall gain with a magnitude of 1000 (i.e. $\left|\frac{v_{out,1}}{v_{\text{antenna,th}}}\right| = 1000$). **Choose values of R_f and R_i for the circuit in (b) to achieve the required gain and to have a Thevenin resistance at $v_{in,1}$ for maximum power.**

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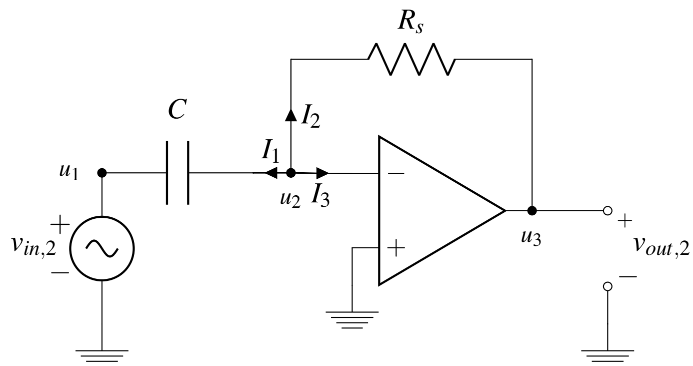
(d) (8 points) Now that we have a signal with a large amplitude, we need to decode it. The trio decides to use FM (like the radios in cars) to send the data, and now they have to decode it. FM is quite complicated, so they enlist EE123 students for help. The EE123 students will do most of the work if Nick, Ryan, and Linda can do one thing first. FM is encoded in such a way that you need to take the derivative of the signal to extract the information. The trio asks Professor Stojanovic for help and he suggests the following circuit:



Let's analyze it in steps.

i. (2 points) **Write the KCL expression at node u_2 in terms of I_1, I_2 and I_3 only.**

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ii. (6 points) **Substitute IV relationships for the components and apply the Golden Rules. Solve for $v_{out,2}$ as a function of R, C and $v_{in,2}$. You may assume negative feedback.**

(e) (6 points) Unfortunately, the above circuit also scales our output by some factor. We **want** an output that is exactly $v_{out} = 1\text{ s} \times \frac{dv_{in,2}}{dt}$. Note that the scaling factor must have units of time for the expression to work out. **Design a circuit whose input is $v_{out,2}$ from (d) and produces an output $v_{out} = 1\text{ s} \times \frac{dv_{in,2}}{dt}$. You may use one op amp and two resistors. The resistors must be given values in terms of C and R_s from part (d).**

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(f) (4 points) Now assume that every block in the diagram (repeated below) has been perfectly implemented.

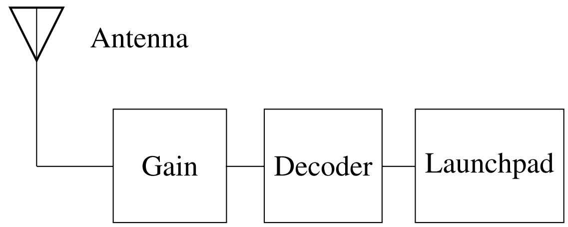

Figure 10.3: Receiver block diagram, repeated

Regardless of your previous answers, assume that Gain = -1000 and the decoder output is $v_{out} = 1\text{ ns} \times \frac{dv_{in}}{dt}$ (nanoseconds because of the time scale of the graph). You are given the input from the antenna as a function of time below. **On the empty graph, plot the output of the entire receiver (the output of part (e), the decoder).**

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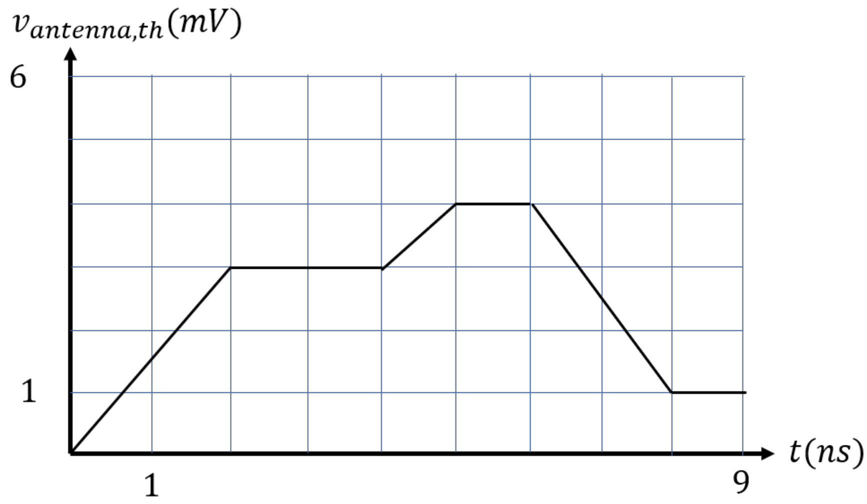
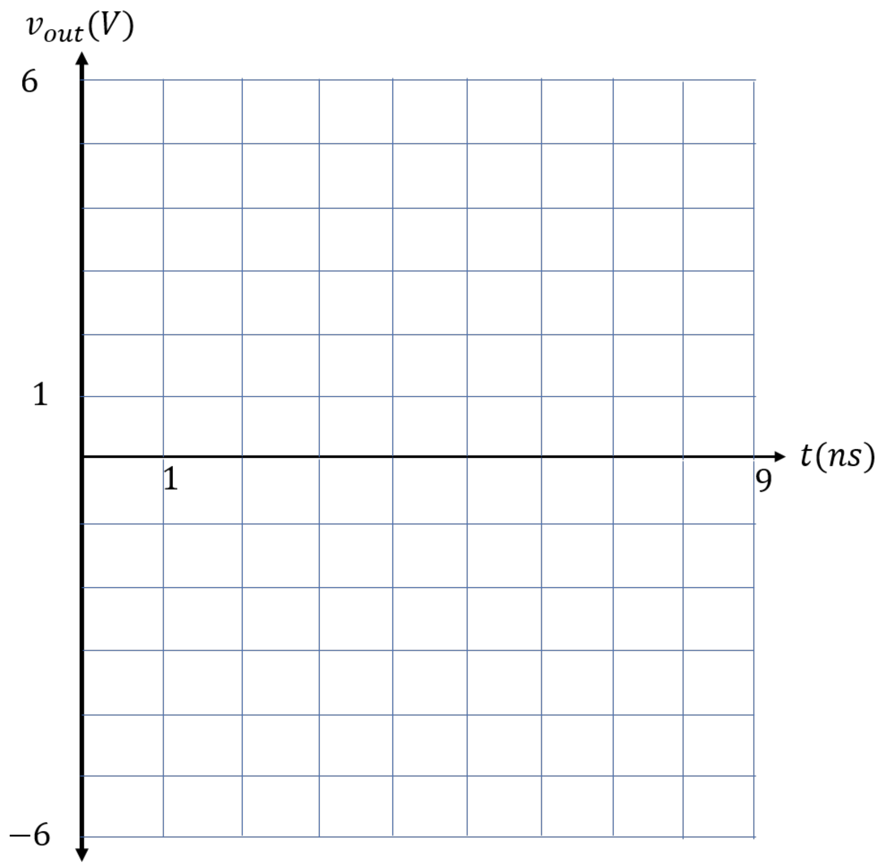
Note: pay attention to the scales on the graphs.





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Extra page for scratchwork.
Work on this page will NOT be graded.

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Doodle page!
Draw us something if you want or give us suggestions, compliments, or complaints.
You can also use this page to report anything suspicious that you might have noticed.

EECS 16A Designing Information Devices and Systems I
Spring 2019 Final Exam Instructions

Read the following instructions before the exam.

There are 10 problems of varying numbers of points. You have 180 minutes for the exam. The problems are of varying difficulty, so pace yourself accordingly, do easier problems first, and avoid spending too much time on any one question until you have gotten all of the other points you can.

There are 46 pages on the exam, so there should be 23 sheets of paper in the exam. The exam is printed double-sided. Do not forget the problems on the back sides of the pages! Notify a proctor immediately if a page is missing. **Do not tear out or remove any of the pages. Do not remove the exam from the exam room.**

No collaboration is allowed, and do not attempt to cheat in any way. Cheating will not be tolerated.

Write your student ID on each page. If a page is found without a student ID, we are not responsible for identifying the student who wrote that page.

You may consult THREE handwritten 8.5" $\times$ 11" note sheets (front and back). No phones, calculators, tablets, computers, other electronic devices, or scratch paper are allowed.

Please write your answers legibly in the boxed spaces provided on the exam. The space provided should be adequate.

In general, show all of your work in order to receive full credit.

Partial credit will be given for substantial progress on each problem.

If you need to use the restrooms during the exam, bring your student ID card, your phone, and your exam to a proctor. You can collect them once you return from the restrooms.

Our advice to you: If you can't solve the problem, state and solve a simpler one that captures at least some of its essence. You might get some partial credit, and more importantly, you will perhaps find yourself on a path to the solution.

Good luck!

Do not turn this page until the proctor tells you to do so.