EECS 16A Designing Information Devices and Systems I
Fall 2019 Midterm 2 - VERSION A

Midterm 2 Solution - VERSION A

PRINT your student ID: ________________________________

PRINT AND SIGN your name: ____________________, ____________________ ____________________
(last name) (first name) (signature)

PRINT your discussion section and GSI(s) (the one you attend): ________________________________

Name and SID of the person to your left: ________________________________

Name and SID of the person to your right: ________________________________

Name and SID of the person in front of you: ________________________________

Name and SID of the person behind you: ________________________________

1. Tell us about something that makes you happy. (1 Point)

2. What courses are you thinking of taking next semester? (1 Point)

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

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3. Power (15 points)

In the circuit shown in Figure 3.1 calculate the total power P_d **delivered** by source I_1 to the rest of the circuit.

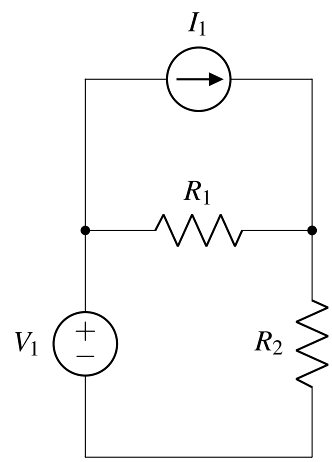
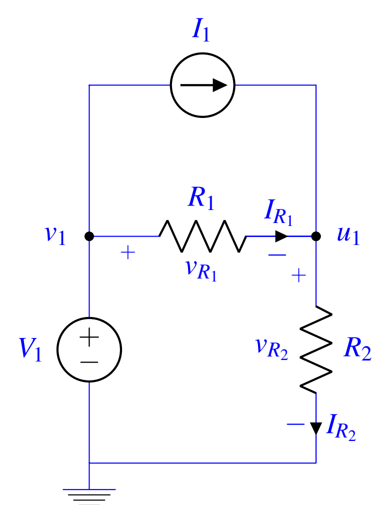


Figure 3.1: Circuit for power calculation

Use the following component values: $R_1 = 2\,\Omega$, $R_2 = 2\,\Omega$, $V_1 = 5\text{V}$, $I_1 = -0.5\text{A}$.

Solution: The labeling steps (steps 1-4) of NVA analysis are shown in the circuit diagram below:



A ground node has been chosen, the node with known voltage is marked as $v_1 = V_1$, the node with unknown voltage is marked as u_1. Also, a direction has been picked for the currents flowing through resistors R_1 and R_2, and the voltage across them is consequently labelled following passive sign convention. We will perform NVA to get the voltage at node u_1, and hence the power delivered to the circuit by the current source I_1. Applying KCL at node u_1:

$$I_1 + I_{R1} = I_{R2}$$

Ohm's law for resistors R_1 and R_2 gives:

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{v_1 - u_1}{R_1} = \frac{V_1 - u_1}{R_1}$$

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{u_1}{R_2}$$

Plugging in the last two equations in the first and solving for u_1 we get:

$$\begin{aligned}
&I_1 + \frac{V_1 - u_1}{R_1} = \frac{u_1}{R_2} \\
\Rightarrow &I_1 R_1 R_2 + V_1 R_2 - u_1 R_2 = u_1 R_1 \\
\Rightarrow &u_1 = \frac{(I_1 R_1 + V_1) R_2}{R_1 + R_2} = 2\text{V}
\end{aligned}$$

So finally the power **dissipated** by I_1 is equal to:

$$P_{I_1} = I_1 V_{I_1} = I_1 (V_1 - u_1) = -0.5\text{A}(5\text{V} - 2\text{V}) = -1.5\text{W}$$

This means that the power delivered to the circuit by I_1 is equal to:
$P_{I_1,\text{del}} = 1.5\text{W}$

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4. Equivalent Circuit (15 points)

Your friend has characterized the circuit shown in Figure 4.1(a) in the lab by first connecting a voltmeter (represented in Figure 4.1(b) by the letter V in a circle) to terminals $A-B$ to measure V_m, then disconnecting the voltmeter and connecting an ammeter (current meter, represented in Figure 4.1(b) by the letter A in a circle) to measure I_m.

Now they ask for your help designing an equivalent circuit model (looking from terminals A and B) consisting only of a current source I_s and resistor R_s.

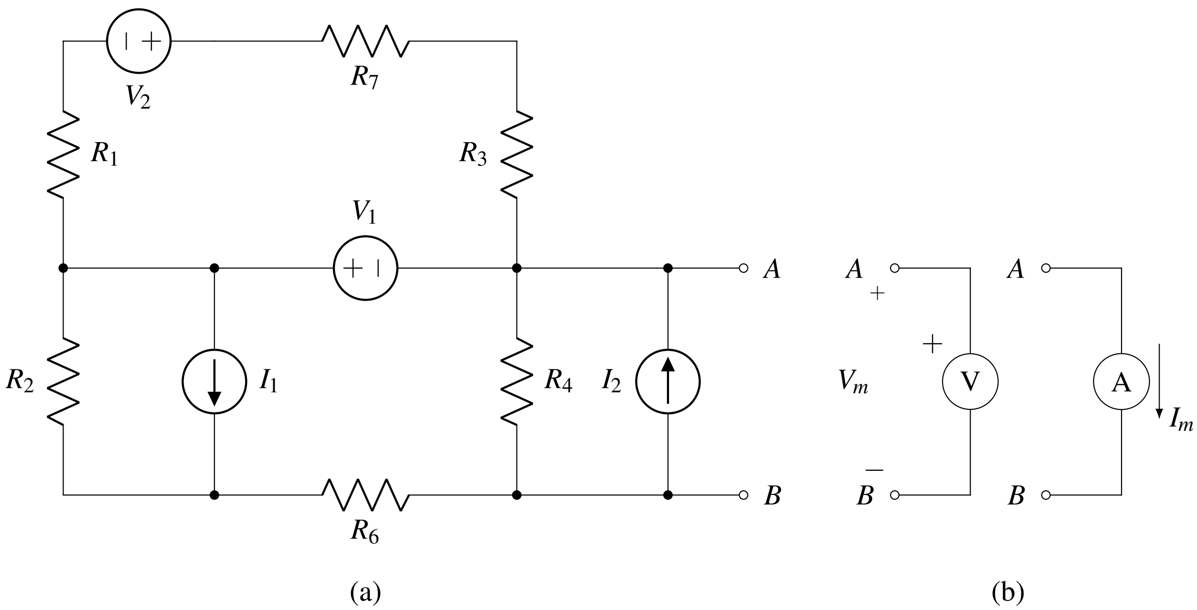

Figure 4.1: (a) Circuit tested in the lab, (b) Circuit used to measure V_m and I_m

Use the following values: $V_m = 6\text{V}$, $I_m = 2\text{A}$.

(a) Draw the I-V characteristic between nodes A, B of the circuit in Figure 4.1(a).

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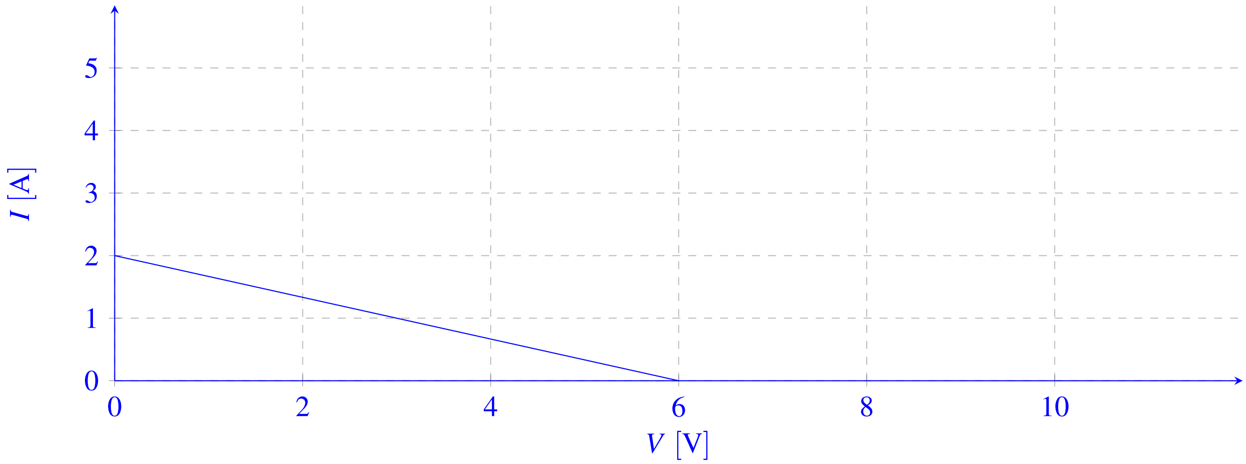
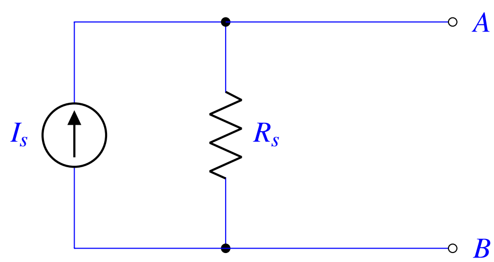
Solution:



(b) Draw the circuit diagram of the equivalent model as seen from nodes A, B. Clearly mark nodes A and B, current I_s (including direction), and resistor R_s.

Solution:

The current meter measures the short circuit current whereas the voltmeter measures the open circuit voltage across terminals A and B. Therefore, using the Norton equivalent model, we can model the circuit of Figure 4.1 (a) as a current source of value $I_s = I_m$ in parallel with a Thevenin resistance $R_s = R_{th} = \frac{V_m}{I_m}$.



(c) Calculate the numerical values of I_s and R_s. Mark your calculated component values in the circuit diagram drawn in part (b). Clearly mark the direction of the current.

Solution: We have $I_s = I_m = 2A$ and $R_s = R_{th} = 3\Omega$.

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5. Next-Phone (15 points)

You have been hired by "Next-Phone", a promising startup that has developed a 3D printer to produce individually customized smartphones.

Only one problem remains: designing accurate position sensing for the printhead. "No problem," you tell your new boss, "I'll take care of that!"

Figure 5.1 shows your design. The printhead is supported by two rollers that move the head in the x direction. Each roller runs on two conductive tracks with resistivities ρ_1 and ρ_2, respectively, length L, and cross-sectional area A. The rollers are made of metal electrically connecting the strips. The printhead is an insulator (i.e. nonconductive material) so it can be modeled as an open-circuit. Roller 1 and Roller 2 are disconnected by the non-conductive Print Head. You connect two voltage sources of voltage V_s as shown in Figure 5.1. You then measure voltage $V_{AB} = V_A - V_B$ to sense position x.

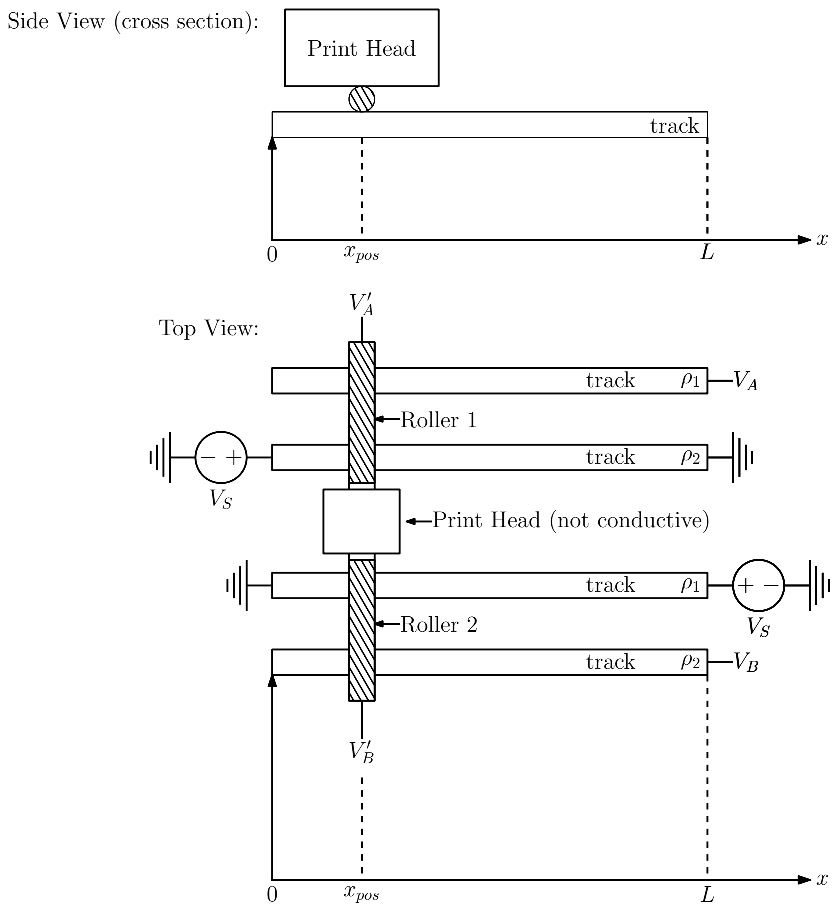

Figure 5.1: 3D printhead position sensor (top and side views)

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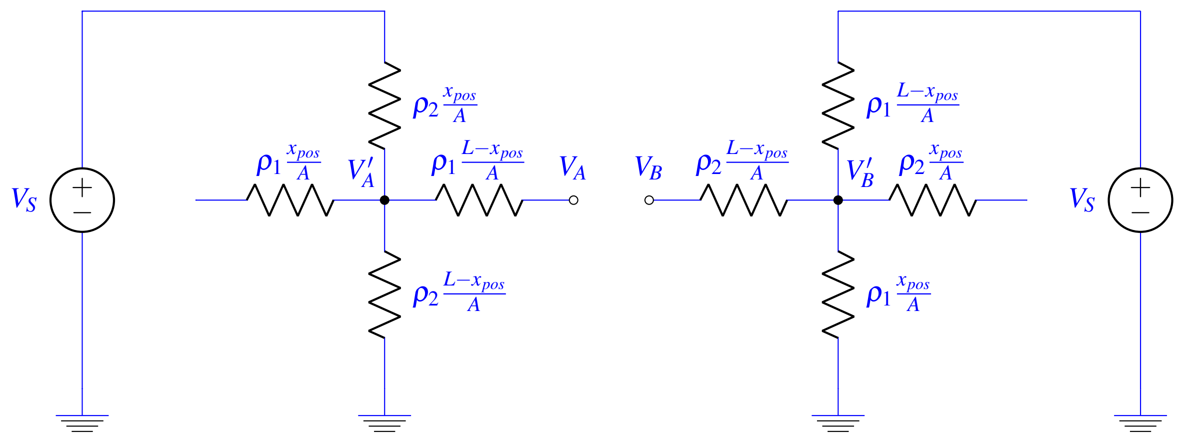
(a) Draw an equivalent circuit diagram of Figure 5.1 consisting of sources, resistors, etc.

Solution: To draw the equivalent circuit diagram, we need to realize that the print head is essentially an open, and that the tracks can be modeled as resistors. So, the equivalent model is:



(b) Derive algebraic expressions for $V_{AA'} = V_A - V_{A'}$ and $V_{BB'} = V_B - V_{B'}$ as a function of x_{pos}.

Solution: Since the resistors connecting nodes A and A' are essentially open we will have that $V_{AA'} = 0$. The same holds for nodes A and A', so $V_{BB'} = 0$.

(c) Find the value of voltage $V_{AB}(x_{pos}) = V_A - V_B$ for $V_s = 10\text{V}$, $\rho_1 = 1\,\Omega\text{m}$, $\rho_2 = 2\,\Omega\text{m}$, $L = 200\text{mm}$, $A = 1\text{cm}^2$, and $x_{pos} = 50\text{mm}$.

Solution: Neglecting the open resistors in our model we only have two voltage dividers, so voltages V_A, V_B are:

$$V_A = \frac{L-x}{L} V_S$$

$$V_B = \frac{x}{L} V_S$$

$$\Rightarrow V_A - V_B = \frac{L-2x}{L} V_S$$

Where we have used the fact that the resistivity and area parameters drop out from the divider equations. Substituting into the derived equation we get that:

$$V_A - V_B = \frac{200\text{mm} - 100\text{mm}}{200\text{mm}} 10\text{V} = 5\text{V}$$

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6. Current Sensor (19 points)

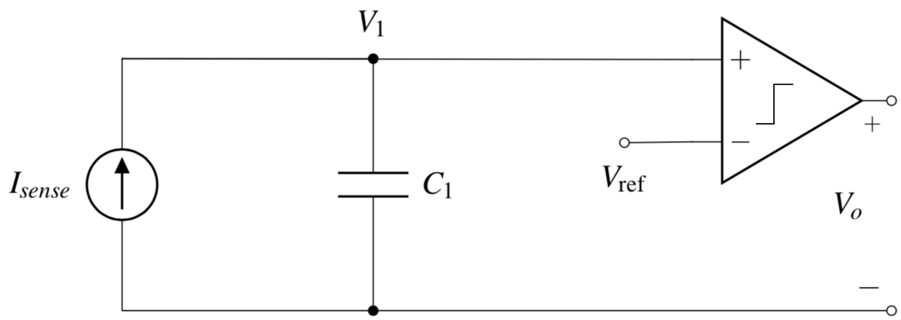

Figure 6.1: First version of the current sensor circuit

(a) You've built a light sensor that outputs a current I_{sense} that is proportional to light intensity. Now you need a circuit that measures this current. Figure 6.1 shows the design of the first version of the current sensor circuit. At time $t = 0$ capacitor C_1 is discharged and a timer (stop watch) is started. When voltage V_1 reaches V_{ref} the comparator "trips" and stops the timer at T_1. The output of the comparator is +1V if $V_1 > V_{ref}$, and 0V otherwise.

In the grids below, plot V_1 and V_o as a function of time t, from $t = 0$ to $t = 2T_1$.

Use $I_{sense} = 0.2\text{A}$, $C_1 = 1\text{F}$, and $V_{ref} = 0.2\text{V}$.

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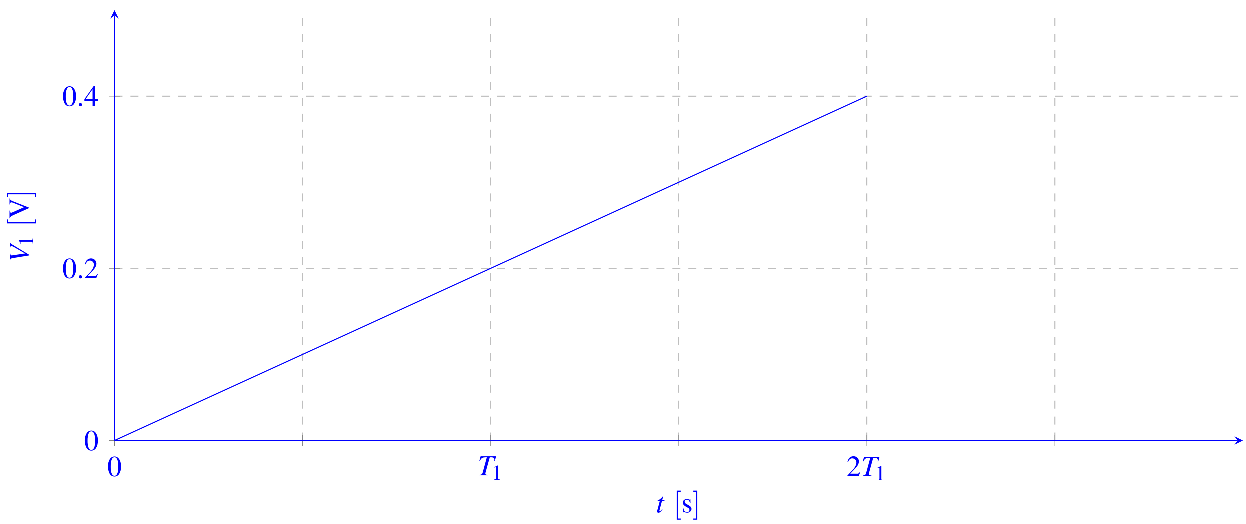
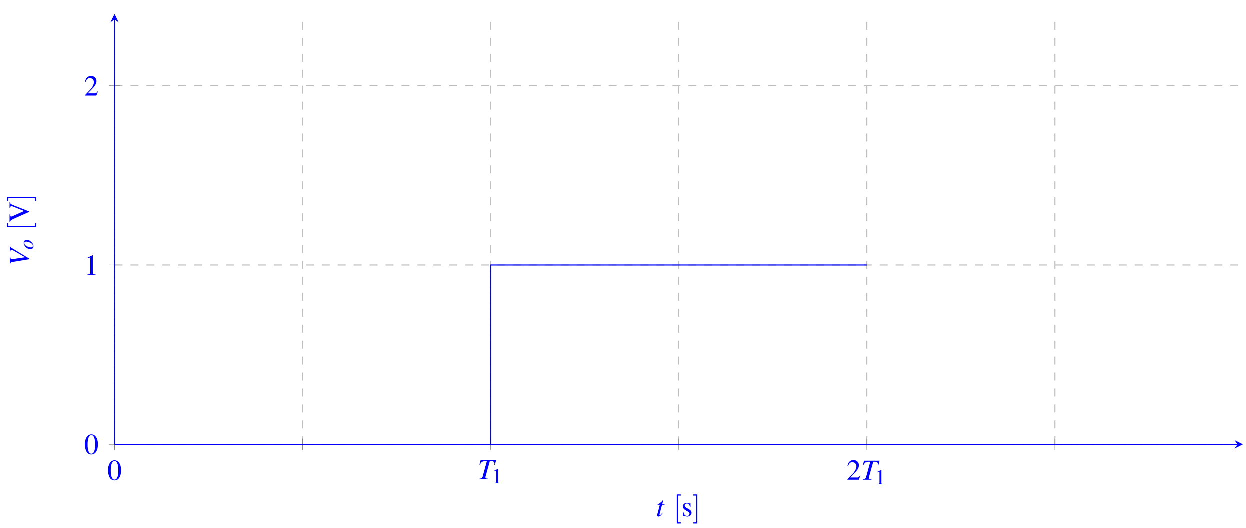
Solution:





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(b) Derive an expression for the time T_1 measured by the timer as a function of I_{sense}, C_1, and V_{ref}. Then, calculate its numerical value using the same component values as given in part (a).

Solution:

Current I_s flows through C_1. $V_1(0) = 0$ because capacitor C_1 is initially uncharged.

$$I_{sense} = C_1 \frac{dV_1(t)}{dt}$$

$$V_1(t) = \int \frac{I_{sense}}{C_1} dt = \frac{I_{sense} t}{C_1} + V_1(0) = \frac{I_{sense} t}{C_1}$$

Time T_1 is the time needed for V_1 to become equal to V_{ref} and the comparator to flip. Therefore,

$$T_1 = \frac{C_1 V_{ref}}{I_{sense}}$$

Plugging in numbers, we get:

$$T_1 = \frac{1F 0.2V}{0.2A} = 1s$$

(c) Figure 6.2 shows an improved current sensor that does not depend on the value of capacitor C_1.

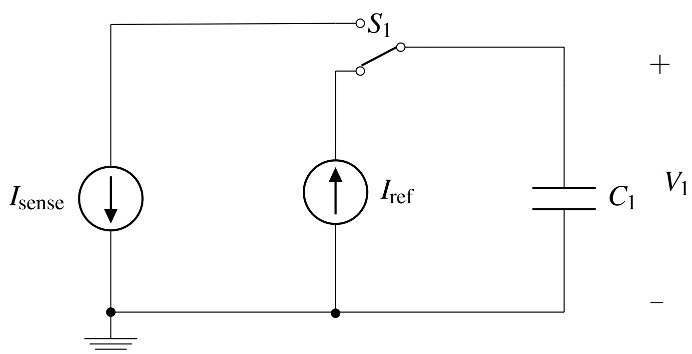

Figure 6.2: Improved Current Sensor

Capacitor C_1 is discharged at time $t = 0$ and switch S_1 is connected to I_{ref}. When V_1 reaches V_{ref} at time T_1, switch S_1 is disconnected from I_{ref} and connected to I_{sense} instead. T_2 is defined as the time from connecting S_1 to I_{sense} to the moment V_1 reaches 0V.

Plot V_1 versus time from $t = 0$ to $t = T_1 + T_2$.

Use $I_{sense} = 0.2$A, $C_1 = 1$F, $V_{ref} = 0.2$V and $I_{ref} = 0.1$A.

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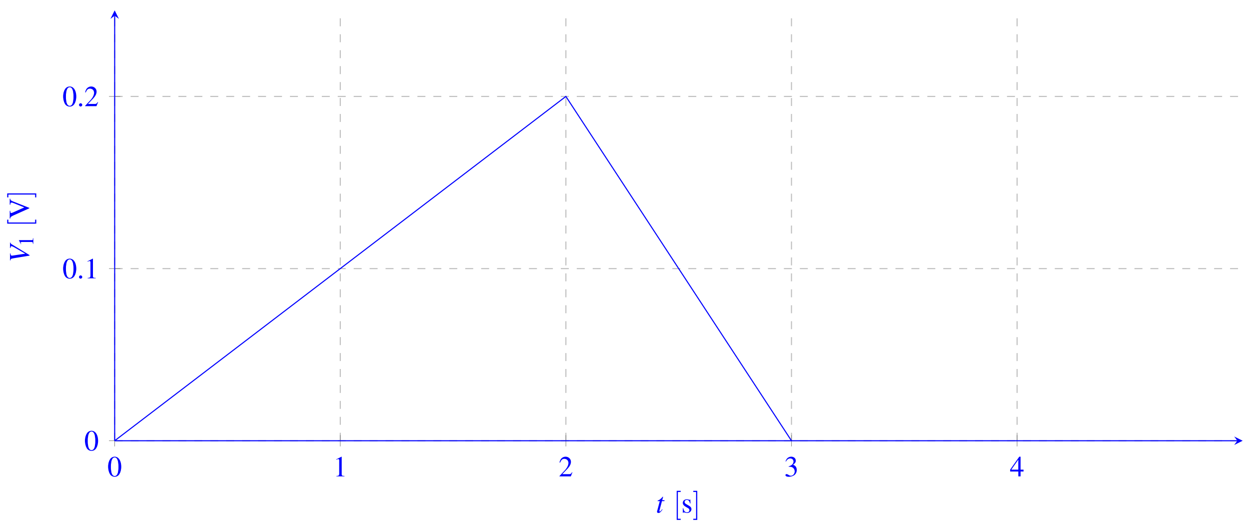
Solution:



(d) Derive an expression for the ratio $\frac{T_1}{T_2}$ (the times calculated in part (c)) as a function of I_{sense}, I_{ref}, C_1, and V_{ref}. Then, calculate its numerical value using the same component values as given in part (c).

Solution:

Initially, capacitance C_1 is charged through the integration of I_{ref}. T_1 is the time needed in order to V_1 to become equal to V_{ref}. Therefore,

$$T_1 = \frac{C_1 V_{ref}}{I_{ref}}$$

T_2 is the time needed for V_1 to get fully discharged or in other words, to go from V_{ref} to 0. Therefore,

$$T_2 = \frac{C_1 V_{ref}}{I_{sense}}$$

The ratio $\frac{T_1}{T_2}$ is found to be:

$$\frac{T_1}{T_2} = \frac{I_{sense}}{I_{ref}}$$

Plugging in numbers, we get:

$$\frac{T_1}{T_2} = 2$$

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7. Capacitor Powered Quadcopter (15 points)

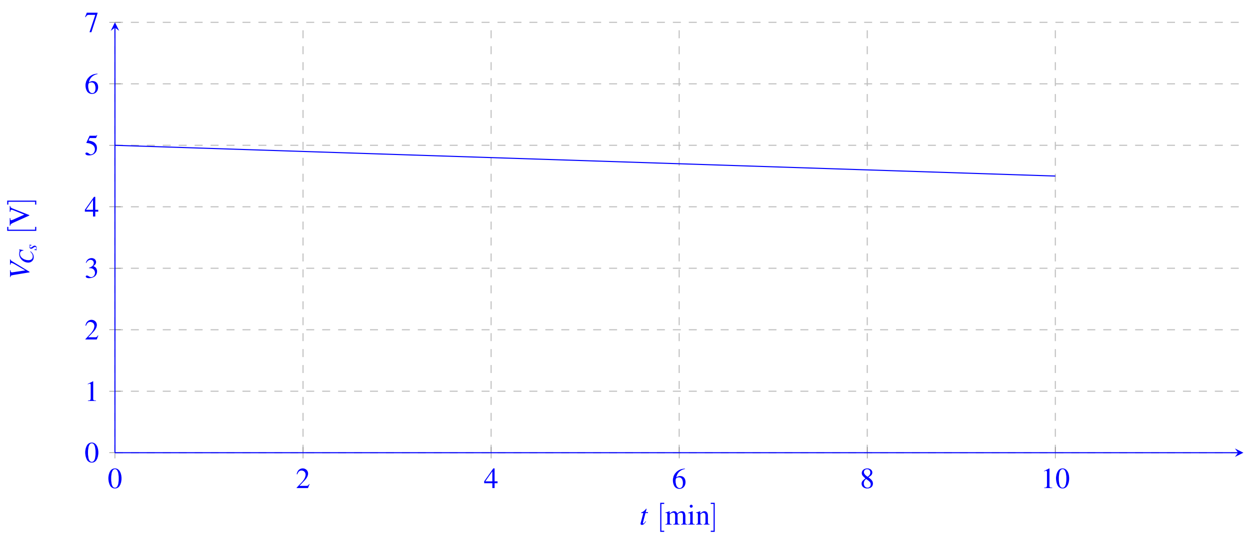
You've made a fun little quadcopter that juggles 10 colorful balls - all while flying in circles above the heads of its stunned audience that cannot get enough of the spectacle.

Unfortunately it takes quite a bit of time to recharge the battery after each demonstration. To shorten the time you decide to replace the battery with a capacitor C_s which can be charged virtually instantaneously.

The drone consumes a constant current $I_d = 0.5$A. The nominal supply voltage of the drone is $V_{nom} = 5$V, but it works with voltages as low as $V_{min} = 4$V (i.e. when the capacitor voltage drops below 4V the drone crashes). V_{nom} is the initial voltage across the capacitor C_s.

(a) Plot the voltage across the capacitor V_{C_s} as a function of time from 0 to 10 minutes in the graph space provided to you below. Use $C_s = 600$F.

Solution:



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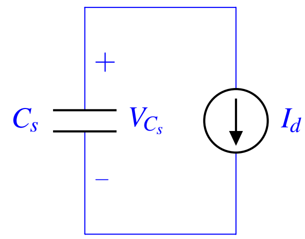
(b) Calculate the minimum value of the capacitor C_s required to support 10 minutes of flying time.

Solution: Since the drone consumes a constant current I_d, our capacitor - drone system can be modeled as:



Hence, the capacitor will be discharged as follows:

$$I_d = C_s \frac{dV}{dt} = C_s \frac{\Delta V}{\Delta t}$$
$$\Delta t = C_s \frac{\Delta V}{I_d} = C_s \frac{V_{nom} - V_{min}}{I_d}$$

Since we want to support a minimum of 10 minute flight we need to have:

$$\Delta t \geq 10\text{min}$$
$$C_s \frac{V_{nom} - V_{min}}{I_d} \geq 10\text{min}$$
$$C_s \geq 10 \cdot 60 \cdot \frac{I_d}{V_{nom} - V_{min}} = 300\text{F}$$

(c) Regardless of your answer in (b), assume $C_s = 5$F. Calculate the ratio of the energy E_2 remaining in the capacitor at the end of the flight divided by the energy E_1 initially stored in the capacitor C_s when it is fully charged.

Solution: The **ratio** of the energy E_2 remaining in the capacitor at the end of the flight divided by the energy E_1 initially stored in the capacitor C_s when it is fully charged does not depend on the actual value of the capacitor but only on the initial and final voltage since it is:

$$\frac{E_2}{E_1} = \frac{\frac{1}{2} C_s V_{min}^2}{\frac{1}{2} C_s V_{nom}^2}$$
$$\frac{E_2}{E_1} = \frac{V_{min}^2}{V_{nom}^2} = \frac{16}{25} = 0.64$$

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8. Fun with charge sharing (19 points)

(a) In Figure 8.1, capacitors C_1 and C_2 are charged to V_1 and V_2 and switch S_1 is open for time $t < 0$. At time $t = 0$, switch S_1 is closed. Calculate V_1 at time $t > 0$.

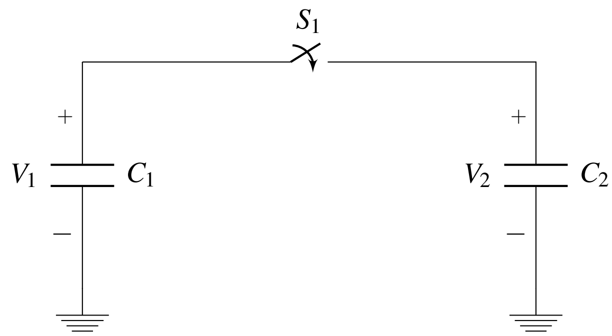

Figure 8.1: Capacitor Charge Sharing

Use the following values: $C_1 = 1\text{F}$, $C_2 = 4\text{F}$, $V_1 = 6\text{V}$, $V_2 = 1\text{V}$.

Solution:

Let us define the initial charge on C_1 as Q_{1i} and the initial charge on C_2 as Q_{2i}. We know that $Q_{1i} = C_1V_{1i}$ and $Q_{2i} = C_2V_{2i}$, where V_{1i} and V_{2i} are the initial voltages across C_1 and C_2, respectively. (i.e. before switch S_1 is closed). We know from conservation of charge that $Q_{1i} + Q_{2i} = Q_{1f} + Q_{2f}$, where Q_{1f} and Q_{2f} are the final charge on C_1 and C_2. (i.e. after switch S_1 is closed). We can write this as:

(1) $\quad C_1V_{1i} + C_2V_{2i} = Q_{1f} + Q_{2f}$.

Additionally, we know that once switch S_1 is closed, the voltage across C_1 and C_2 must be the same, because they are now in parallel with each other. Specifically, $V_{1f} = V_{2f}$ where V_{1f} and V_{2f} are the final voltages across C_1 and C_2, respectively. (i.e. after switch S_1 is closed). Therefore,

$C_1V_{1i} + C_2V_{2i} = (C_1 + C_2)V_1 f$

At time $t > 0$, $V_{1f} = V_1 = \frac{C_1V_{1i}+C_2V_{2i}}{C_1+C_2}$

Plugging in numbers, we get:

$V_{1f} = V_1 = 2V$

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(b) The circuit shown in Figure 8.2 operates in two phases. During phase 1, switches labeled S_1 are closed and switches S_2 are open. During phase 2, switches S_1 are open and switches S_2 are closed, as illustrated in the timing diagram shown in Figure 8.3.

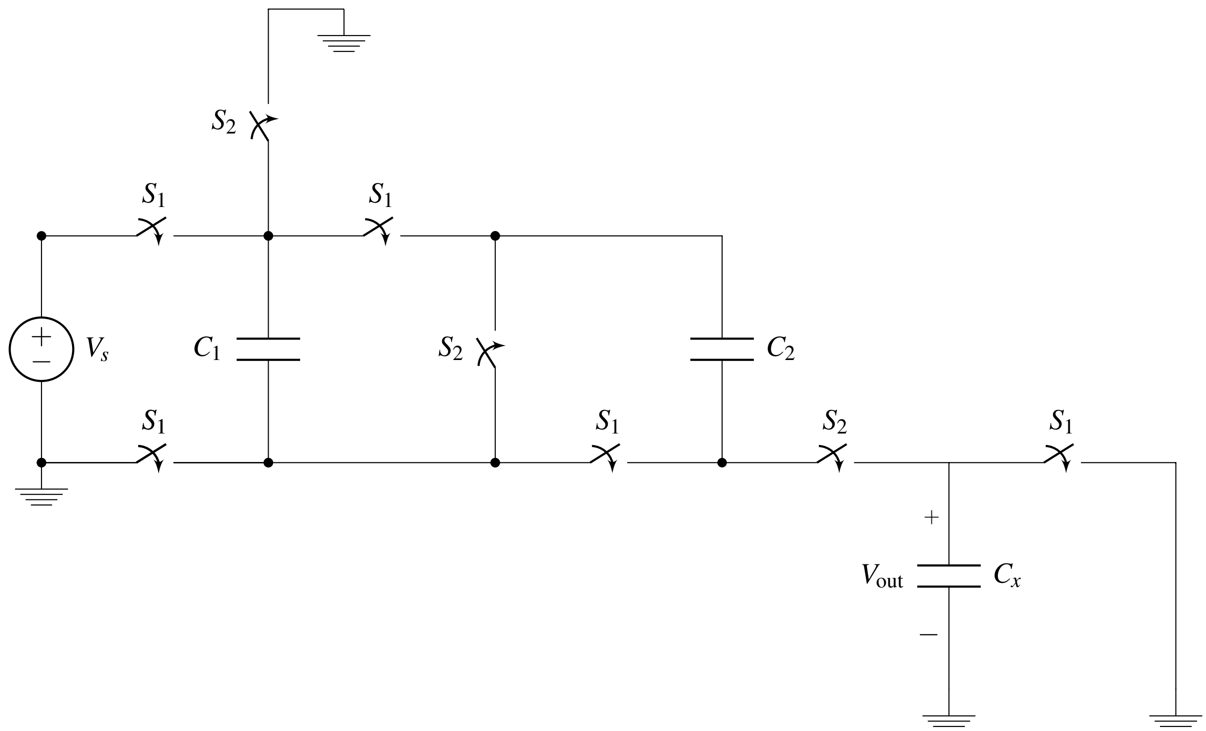

Figure 8.2: Capacitor Charge Sharing

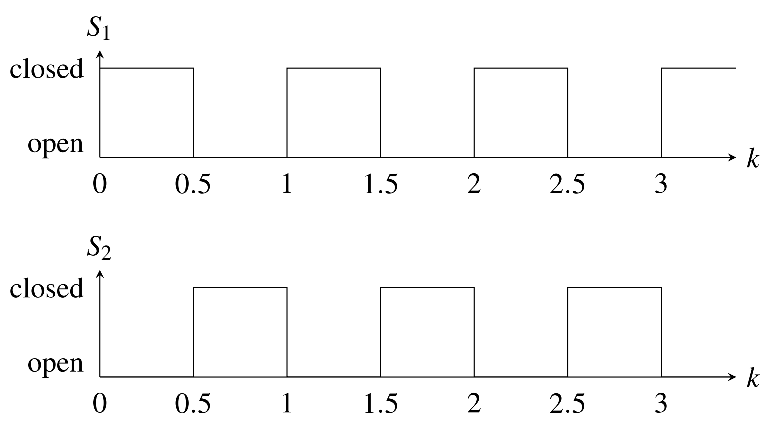

Figure 8.3: Timing diagram for switches

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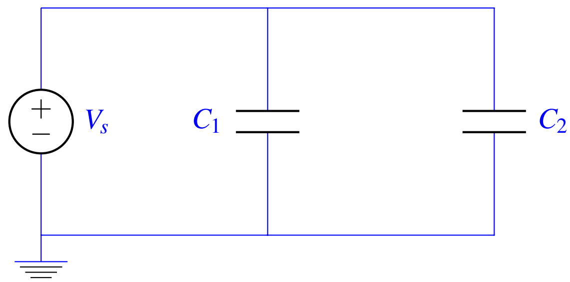
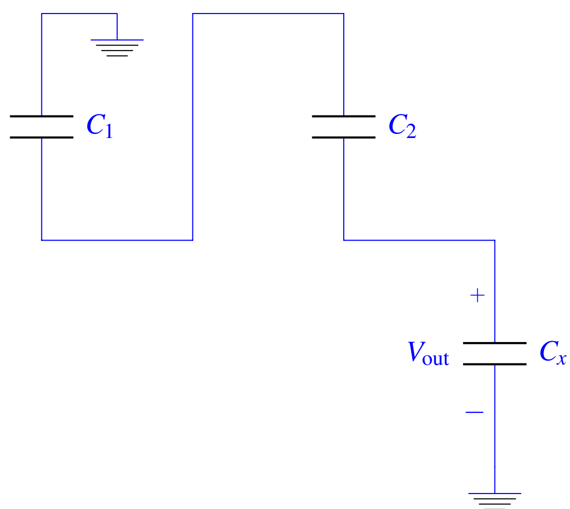
i. Redraw the circuit during phase 1. Replace closed switches with "wires" and open switches with "open circuits" (i.e. just omit them from the diagram). Use $C_1 = C_2 = C_0$.
Solution:



ii. Redraw the circuit during phase 2. Replace closed switches with "wires" and open switches with "open circuits" (i.e. just omit them from the diagram). Use $C_1 = C_2 = C_0$.
Solution:



iii. Calculate the value of the voltage V_{out} during phase 2 as a function of C_0, C_x, and V_s.
Solution:
In phase 1, the total charge is equal to $Q_1 + Q_2 = -C_1 V_s + C_2 V_s$.
In phase 2, from charge conservation, we get:

$$-C_1 V_s + C_2 V_s = C_1 V_2 + C_2 (V_2 - V_{out}) \tag{1}$$

Also, initially, the charge at the bottom plate of C_2 was $-C_2 V_s$. Therefore, from charge conservation, we get:

$$-C_2 V_s = C_2 (V_{out} - V_2) + C_x V_{out} \tag{2}$$

$$V_2 = \frac{(C_x + C_2) V_{out} + C_2 V_s}{C_2} \tag{3}$$

Plugging (3) into (1),

$$V_{out} = \frac{-2 C_1 C_2 V_s}{C_1 C_x + C_1 C_2 + C_2 C_x} = \frac{-2 V_s}{1 + \frac{2 C_x}{C_0}}$$