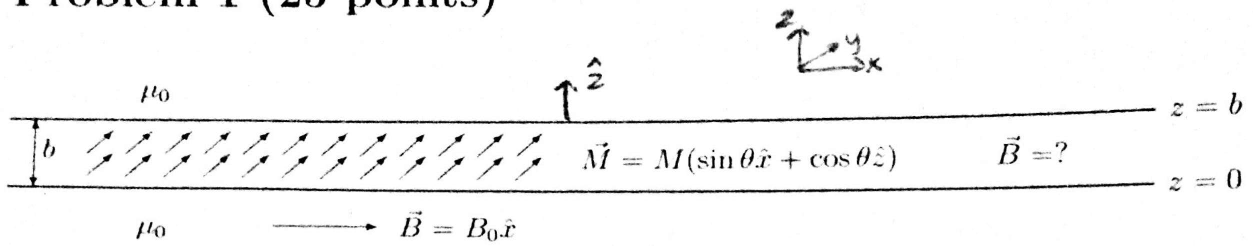
Problem 1 (25 points)



An infinite slab of a ferromagnet has uniform magnetization $\vec{M}$.
The thickness of the ferromagnet is b and it fills the space $0 < z < b$.
The magnetization of the ferromagnet is at an angle θ to the z-axis and is given by

$$\vec{M} = M(\sin\theta\hat{x} + \cos\theta\hat{z}).$$

We are also given that the magnetic field is $\vec{B} = B_0\hat{x}$ below the ferromagnet ($z < 0$).
What is the magnetic field inside the ferromagnet ($0 < z < b$)?

Note: the slab fills the whole $-\infty < x, y < \infty$ range, the magnetic field is time independent, and there is no electric field.

Problem 2 (25 points)

The electric field in free space (devoid of any charges or currents) is given by

$$\vec{E} = E_0\cos(kz - \omega t)\hat{x} + E_1\sin(kz - \omega t)\hat{y}, \qquad \text{with } \omega = kc,$$

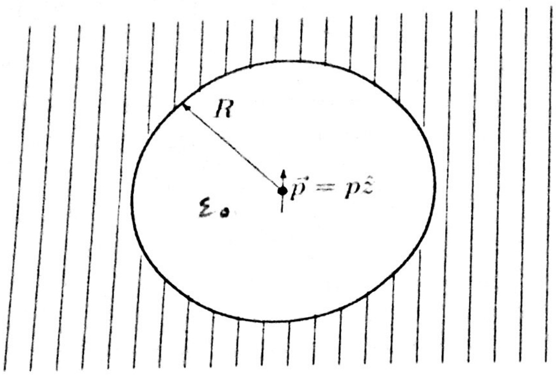
where E_0, E_1 and k are given constants.

(a) Calculate the magnetic field $\vec{B}$.

(b) Calculate the total electromagnetic energy that passes through a square of area A parallel to the xy-plane during a time period $T = 2\pi/\omega$.

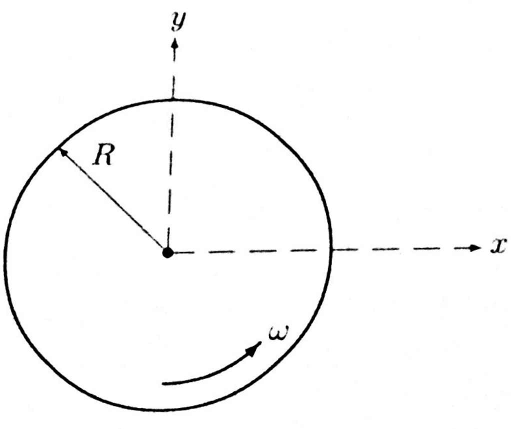
Note: Technically, there are many answers to part (a), and hence part (b), since we can always add an arbitrary magnetostatic field to the solution. Therefore, you can pick any answer that is consistent with the laws of physics, but it's easier to pick the simplest solution for $\vec{B}$.

Problem 3 (25 points)



A conductor has a cavity in the shape of a ball of radius R. The cavity is empty (with electric permittivity ϵ_0), except for a perfect electric dipole, with dipole moment $\vec{p} = p\hat{z}$, at the center. Find the electrostatic potential V inside the cavity, assuming that the conductor is grounded (at $V = 0$). (You can express your results in spherical coordinates.)

Problem 4 (25 points)



A thin disk of radius R is lying on the xy-plane, centered at the origin. It carries a uniform surface charge density σ (with total charge $Q = \pi R^2 \sigma$) and is spinning with angular velocity ω (around the z-axis). Find an expression for that magnetic field $\vec{B}$ far away from the disk (at distance $r \gg R$).

Note: you can neglect terms that fall off like $O(\frac{1}{r^4})$, and can express your answer either as $\vec{B}(x, y, z)$ in Cartesian coordinates, or as $\vec{B}(r, \theta, \phi)$ in spherical coordinates, whichever you prefer.