

NAME:

ID # :

# 1	# 2	# 3	# 4	# 5	#6	#7	TOTAL
8	15	12	12	10	8	12	77

Instructions:

- 1 Write your name and student ID number.
- 2 Read the questions carefully.
- 3 This exam has 7 questions worth 76 points.
- 4 Please write your solution clearly.

Problem # 1 ($1 \times 8 = 8$ points)

For each statement, state whether the claim is True or False.

Circle your answer. No explanation is necessary.

–1 **points for incorrect answers** so guessing is not advised.

- 1 **True or False** First-order linear systems can have an oscillatory free-response.
- 2 **True or False** If two square matrices A and B have the same eigenvalues, then $A = B$.
- 3 **True or False** Proportional control can always stabilize a first-order LTI plant.
- 4 **True or False** Anti-wind up strategies are needed when using **pure** proportional control.
- 5 **True or False** If you have a bad plant model, feedback linearization is a bad idea.
- 6 **True or False** Increasing the proportional gain k_p , reduces the time constant.
- 7 **True or False** Suppose the realization $\Sigma(A, B, C, D)$ is stable.. Then A is invertible.
- 8 **True or False** The steady-state response of a stable linear system due to a sinusoidal input depends on the initial conditions.

Problem # 2 (2 +2+2+3+3+3 = 15 points)

- (a) Give any two reasons why state-space methods are very powerful.

Reason 1:

Reason 2:

- (b) What is the single most important reason to use integral control?

Answer:

- (c) When should we use anti-windup control strategies?

Answer:

- (d) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} p & 1 & 0 \\ 0 & q & 0 \\ 1 & 1 & r \end{bmatrix}$$

$\lambda_1 =$

$\lambda_2 =$

$\lambda_3 =$

- (e) Calculate $\sin(A)$ for the matrix

$$A = \begin{bmatrix} \pi & 100 \\ 0 & 2\pi \end{bmatrix}$$

Answer:

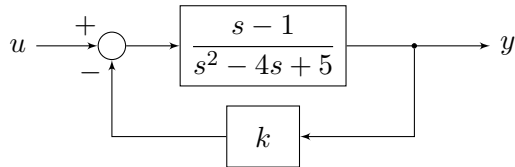
- (f) Find the DC gain matrix of the transfer function

$$P(s) \sim \left[\begin{array}{cc|ccc} -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array} \right]$$

Answer:

Problem # 3 (4 + 4 + 4 = 12 points)

(a) Consider the feedback system shown below.



For what values of k is this feedback system stable?

Answer:

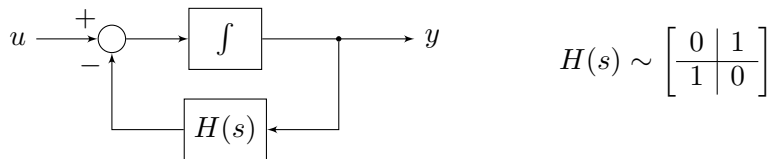
(b) Consider the plant with transfer function

$$H(s) = \frac{s^2 + cs + d}{s^2 + 4s + 4}$$

We apply the input $u(t) = \sin(2t)$. The steady-state response is zero. Find c and d .

$c =$ $d =$

(c) Consider the feedback system shown below



$$H(s) \sim \left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} \right]$$

Find the transfer function from u to y .

Answer:

Problem # 4 (12 points)

Below are 6 different realizations.

A. $\left[\begin{array}{cc|c} 0 & 1 & 0 \\ -8 & -2 & 1 \\ \hline 8 & 0 & 0 \end{array} \right]$

B. $\left[\begin{array}{c|c} -1 & 1 \\ \hline 1 & 0 \end{array} \right]$

C. $\left[\begin{array}{cc|c} 0 & 1 & 0 \\ -2 & -2 & 1 \\ \hline 2 & -4 & 0 \end{array} \right]$

D. $\left[\begin{array}{cc|c} 0 & 1 & 0 \\ -64 & -8 & 1 \\ \hline 64 & 0 & 0 \end{array} \right]$

E. $\left[\begin{array}{c|c} -1 & 1 \\ \hline -1 & 0 \end{array} \right]$

F. $\left[\begin{array}{cc|c} 0 & 1 & 0 \\ -2 & -2 & 1 \\ \hline 0 & 1 & 0 \end{array} \right]$

The unit-step response, starting from zero initial conditions at $t = 0$ for these models are shown on the next page. Match these step responses with the models by entering the letters **A** through **F** in the boxes provided.

No partial credit. No explanations are necessary.

Correct answers get 2 points, incorrect answers get -1 points.

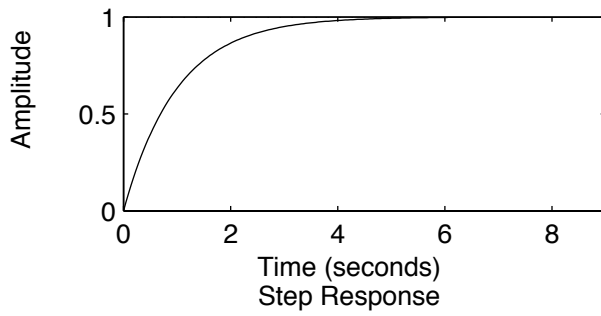
1.	4.
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2.	5.
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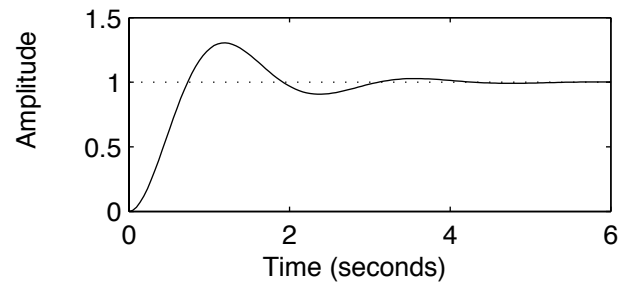
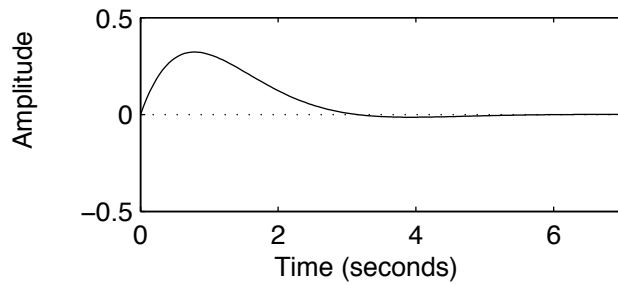
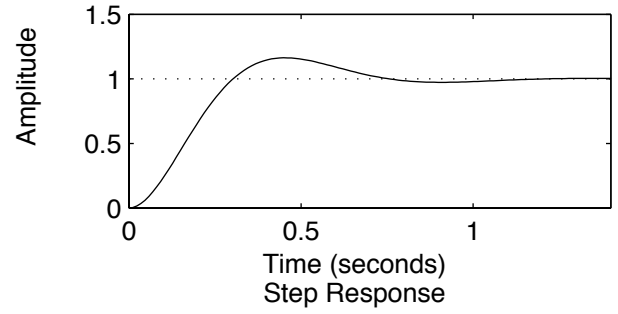
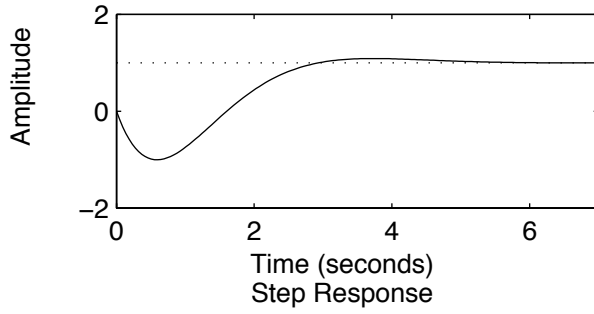
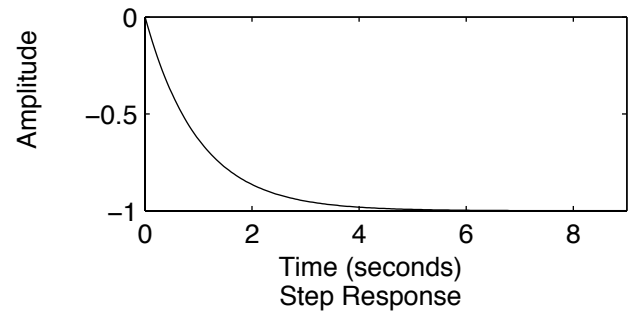
3.	6.
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1.	4.
2.	5.
3.	6.

Step Response



Step Response



Problem # 5 (5+5 = 10 points)

Please show your work to receive full credit.

(a) Consider the feedback system shown below.

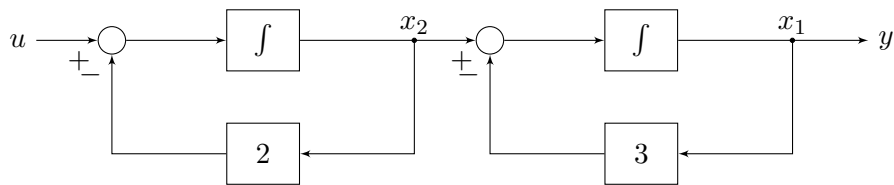
Find a realization for the transfer function $H(s)$ from u to y , i.e. find matrices A, B, C, D such that

$$H(s) \sim \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$

Use the state variables

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where x_1 and x_2 are the signals shown.



$A =$

$B =$

$C =$

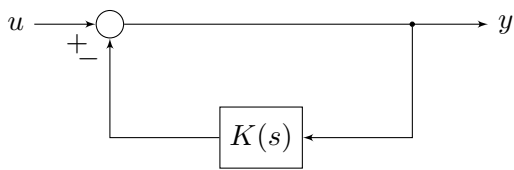
$D =$

- (b) Consider the feedback system shown below.
Suppose $K(s)$ has state-space realization

$$K(s) \sim \left[\begin{array}{c|c} F & G \\ \hline H & 0 \end{array} \right]$$

Compute any state space realization for the transfer function $H(s)$ from u to y , i.e. find matrices A, B, C, D such that

$$H(s) \sim \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$



$A =$	$B =$	$C =$	$D =$
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Problem # 6 (2 + 2 + 2 + 2 = 8 points)

Consider the LTI system with input u and output y modeled by

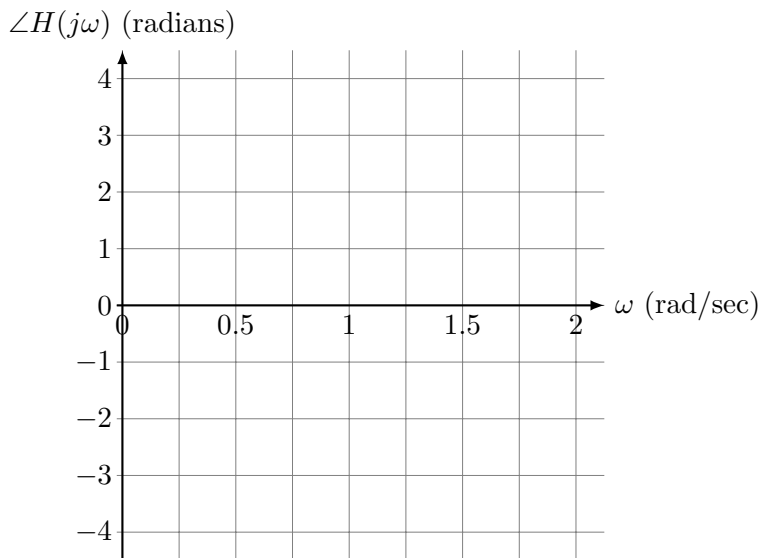
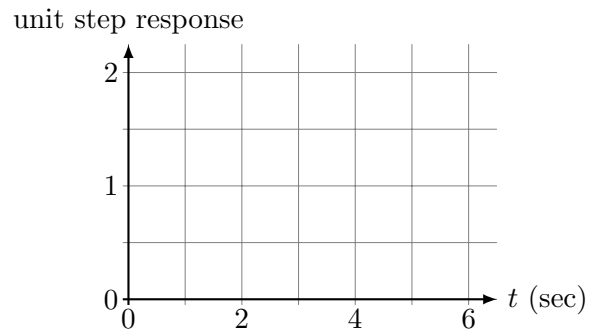
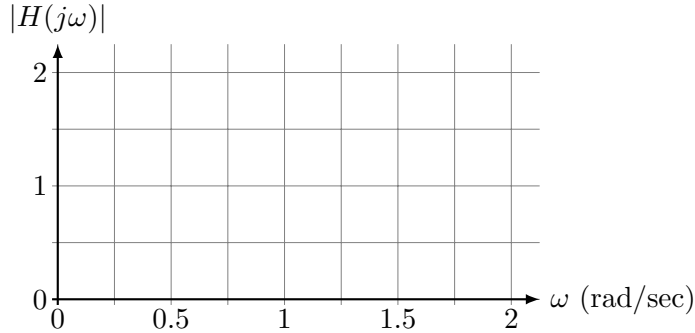
$$y(t) = u(t - 2)$$

- (a) Find the transfer function $H(s)$ from u to y .
- (b) Plot the unit step response.
- (c) Plot the magnitude frequency response of $H(s)$.
- (d) Plot the phase frequency response of $H(s)$.

Use the graph sheets provided below.

To make it easier, the scales are linear (not log-frequency or decibels).

No partial credit. No explanations are necessary.

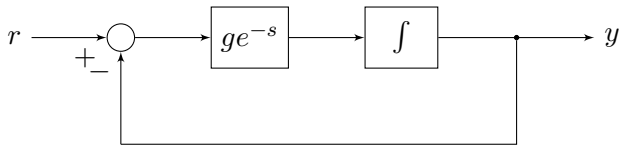


$H(s) =$

Problem # 7 (12 points)

For what values of g is the feedback system shown below stable?
Your answer should be in the form

$$g_{\min} < g < g_{\max}$$



$g_{\min} =$

$g_{\max} =$