Spring 2008 MSE111

Midterm Exam

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80 minutes, 90 points total, 10 pages

Name: Solutions

SID: ____________________

Problem	1	2	3	4	5	6	TOTAL
Points Possible	8	18	10	19	19	16	90
Points							

SHOW ALL OF YOUR WORK!!!

Answers given without supporting calculations will be marked wrong, even if they are numerically correct.

1. (8 pts.) True/False
CIRCLE *T* or *F* indicating whether the statement is true or false

1) Acoustic phonons have longer wavelengths than optical ones for any given material.
T F

2) The effective electron mass is directly proportional to $\frac{\partial^2 E}{\partial k^2}$.
T F

3) The product of electron concentration and hole concentration (at a given temperature) is constant for intrinsic semiconductors, but may vary when the semiconductor is doped.
T F

Solutions

4) The bonding of SiC is completely covalent in nature.
T F

5) At a given velocity, an electron's wavelength is longer than a neutron's wavelength.
$\lambda = \frac{h}{mv}$ $m_e < m_n$
T F

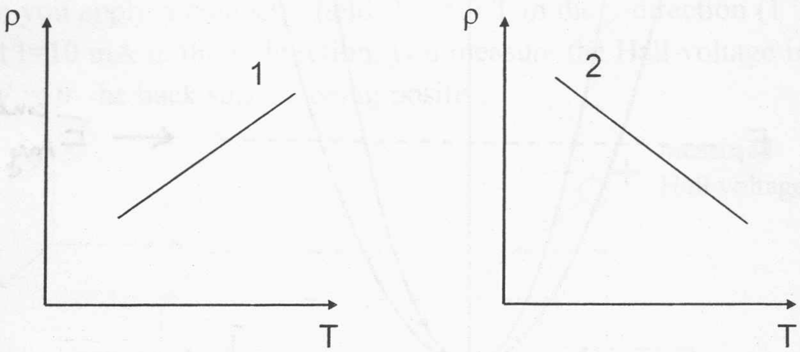
6) For indirect gap semiconductors, photon absorption across the indirect bandgap must be phonon assisted.
T F

7) What is the room temperature value of k_BT? ~25 meV

8) Metals have bands of forbidden energy.
T F

2. (18 pts) Electrical Properties of Metals and Semiconductors

The resistivity vs. temperature is plotted below for two unknown materials, labeled 1) and 2).



a) (3 pts.) Would you guess that material 1) is a metal or an intrinsic semiconductor? Why does the resistivity increase with temperature?

- metal
- ρ increases with temperature due to phonon scattering

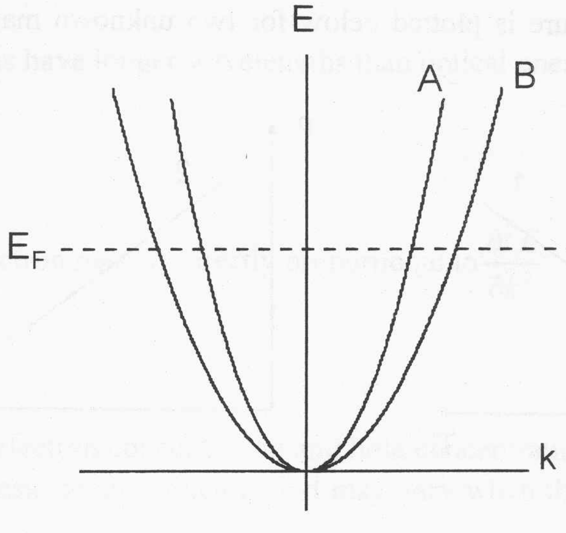
b) (3 pts.) Would you guess that material 2) is a metal or an intrinsic semiconductor? Why does the resistivity decrease with temperature?

- intrinsic semiconductor
- ρ decreases with temperature because the carrier concentration is increasing exponentially

c) (3 pts.) At room temperature the resistivity of Ni is ~60 nΩm and the resistivity of Cu is ~15 nΩm. Would you expect the resistivity of a Ni-Cu alloy with 98% Ni and 2% Cu to have a greater or smaller resistivity than pure Ni and why?

- greater
- ρ increases due to alloy scattering or impurity scattering

Part of the dispersion relation is plotted below for two different materials, labeled A) and B). The Fermi level is also indicated.



$\leftarrow E_{Avg.}^{conduction\ e^-}$

d) (3 pts.) Are these materials behaving like metals or like semiconductors and why?

- metals
- E_F is within a band of allowed states. Empty states are immediately adjacent to filled ones.

e) (4 pts.) Based on the above plot which material, A or B, would have a higher electron mobility and why?

- $\mu_A > \mu_B$

$\mu \propto \frac{1}{m^*}$

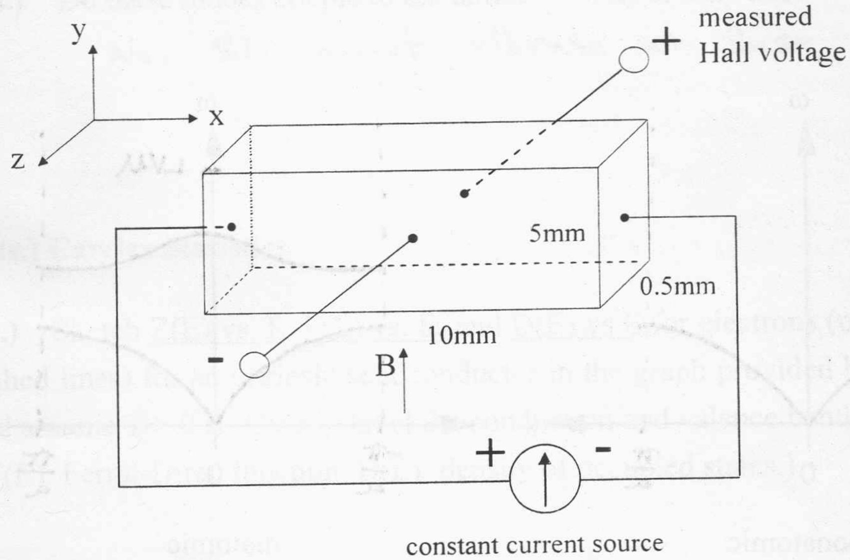
$m^* \propto \left(\frac{\partial^2 E}{\partial k^2}\right)^{-1} \rightarrow \boxed{\mu \propto \frac{\partial^2 E}{\partial k^2}}$

f) (2 pts.) Indicate on the above diagram the average energy of electrons contributing to conduction?

$E_{Avg.}^{conduction\ e^-} \simeq E_F$

3. (10 pts.) Hall Effect

You make a Hall Effect measurement of a GaAs sample (10 x 5 x 0.5 mm^3) with magnetic field at right angles to the bias voltage and the measured Hall voltage, as shown below. When you apply a magnetic field B = 5.0 T in the y-direction (1 Tesla = 1Vs/m^2) and a current I=10 mA in the x-direction, you measure the Hall voltage in the z-direction to be 200 mV with the back surface being positive.



a) (6 pts.) Determine the free carrier type and concentration in cm^{-3}.

$\boxed{n\text{-type}}$

$$J = \frac{0.010\ A}{(5\times10^{-3} m)(0.5\times10^{-3} m)} = 4{,}000\ A/m^2$$

$$E_H = \frac{V_H}{w} = \frac{JB}{ne} \qquad E_H = \frac{0.2V}{0.5\times10^{-3} m} = 400\ V/m$$

$$n = \frac{JB}{eE_H} = \frac{(4000\ A/m^2)(5.0\ Vs/m^2)}{(1.6\times10^{-19} C)(400\ V/m)} = 3.13\times10^{20}\ m^{-3} = \boxed{3.13\times10^{14}\ cm^{-3}}$$

b) (4 pts.) You now use a voltmeter (not shown) to measure a voltage of 3.0V across the sample in the x direction. Determine the resistivity (in Ω-cm) and mobility (in cm^2/Vs) of the GaAs sample.

$$R = \frac{V}{I} = \frac{3.0V}{0.010A} = 300\ \Omega \qquad R = \rho\frac{L}{A} \rightarrow \rho = \frac{RA}{L}$$

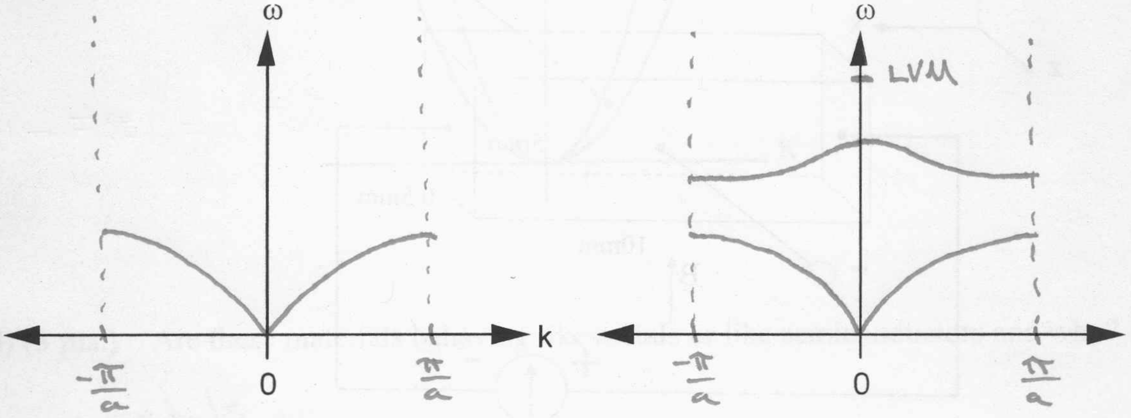
$$\rho = \frac{(300\Omega)(5\times10^{-3} m)(0.5\times10^{-3} m)}{10\times10^{-3} m} = 0.075\ \Omega m = \boxed{7.5\ \Omega\text{-cm}}$$

$$\sigma = \frac{1}{\rho} = n\mu e$$

$$\mu = \frac{1}{\rho n e} = \frac{1}{(7.5\ \Omega cm)(3.13\times10^{14} cm^{-3})(1.6\times10^{-19} C)} \simeq \boxed{2660\ cm^2/Vs}$$

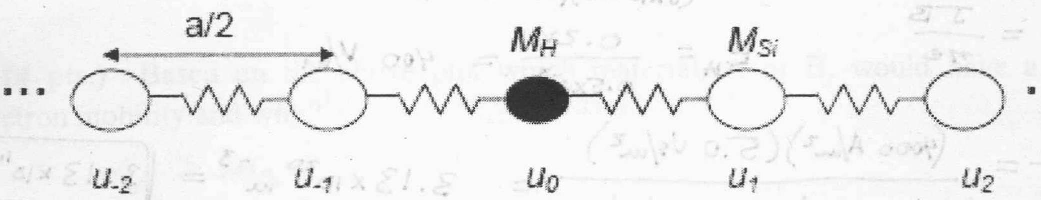
4. (19 pts.) Bonding and phonons

a) (8 pts.) Sketch the phonon dispersion relation for the first Brillouin zone of a monatomic and a diatomic lattice. Label the branches (only draw the longitudinal modes), and the limits of the x-axis. Use "a" as the lattice constant.



monatomic diatomic

b) (6 pts.) Now consider a linear chain of Si atoms with one H impurity (pictured below).



Write the equations of motion for the H atom and a Si atom. Assume only nearest-neighbor interactions and that the force constants (C) are the same between all atoms. Where u_x is the amplitude of the displacement for position x.

$$M_H \frac{\partial^2 u_0}{\partial x^2} = C(u_1-u_0) - C(u_0-u_{-1}) = C(u_1+u_{-1}-2u_0)$$

$$M_{Si} \frac{\partial^2 u_1}{\partial x^2} = C(u_2-u_1) - C(u_1-u_0) = C(u_2+u_0-2u_1)$$

c) **(2 pts.)** How will the frequency of the local vibrational mode (LVM) compare to the frequency range of an unperturbed linear chain of Si atoms? Mark this LVM on the **appropriate** plot from part a.

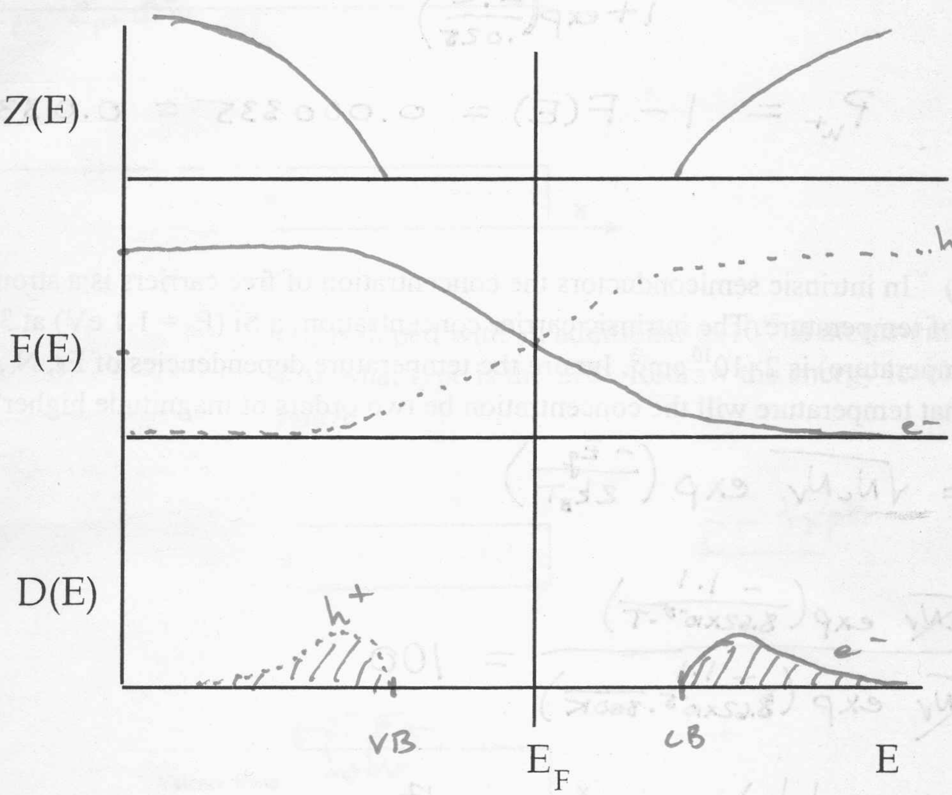
$\omega_{LVM} >> \omega_{Si}^{optical}$

d) **(3 pts.)** Do these modes couple to the lattice? Why or why not?

No. Si cannot vibrate at these frequencies.

5. (19 pts.) Carrier Statistics

a) (8pts.) Sketch Z(E) vs. E, F(E) vs. E, and D(E) vs E for electrons (use solid lines) and holes (use dashed lines) for an intrinsic semiconductor in the graph provided below. Use a common E axis, and assume T > 0 K. Clearly label the conduction and valence band edges. (Z(E): density of states, F(E): Fermi-Dirac function, D(E): density of occupied states.)



b) (3 pts.) For intrinsic semiconductors the Boltzmann tail approximation is typically used in which $F(E) \approx \exp(-\frac{E-E_F}{kT})$. Show how this approximation is valid.

For $E - E_F >> kT$, $\exp\left(\frac{E-E_F}{k_BT}\right) >> 1$

so $$F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{k_BT}\right)} \simeq \frac{1}{\exp\left(\frac{E-E_F}{k_BT}\right)}$$

c) (2 pts.) If the probability of finding an electron in a certain state is 15%, what is the probability of finding a hole in that state?

$1 - F(E) = 85\%$

d) (3 pts.) What is the probability of finding a hole 0.2eV below the Fermi energy at room temperature?

$$F(E) = \frac{1}{1+\exp\left(\frac{-.2}{.025}\right)} = 0.99966$$

$$P_{h^+} = 1 - F(E) = 0.000335 = 0.0335\%$$

e) (3 pts.) In intrinsic semiconductors the concentration of free carriers is a strong function of temperature. The intrinsic carrier concentration in Si (E_g = 1.1 eV) at 300 K (room temperature) is 2×10^{10} cm^{-3}. Ignore the temperature dependencies of E_g, N_c, and N_v. At what temperature will the concentration be two orders of magnitude higher?

$$n_i = \sqrt{N_cN_v}\exp\left(\frac{-E_g}{2k_BT}\right)$$

$$\frac{\cancel{\sqrt{N_cN_v}}\exp\left(\frac{-1.1}{8.62\times10^{-5}\cdot T}\right)}{\cancel{\sqrt{N_cN_v}}\exp\left(\frac{-1.1}{8.62\times10^{-5}\cdot 300K}\right)} = 100$$

$$\exp\left(\frac{-1.1}{k_BT}\right) = 3.36\times10^{-17}$$

$$\frac{-1.1}{k_BT} = -37.93 \longrightarrow \boxed{T = 336\ K}$$

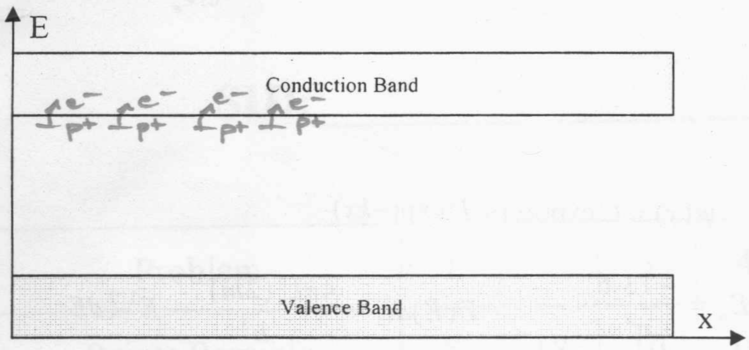
6. Extrinsic Semiconductors (16 pts.)

a) (6 pts.) Predict the behavior of the following impurities in Si and GaAs. Use the given terms to label each case.

Terms:
- Donor
- Acceptor
- Double Donor
- Double Acceptor
- None of the Above

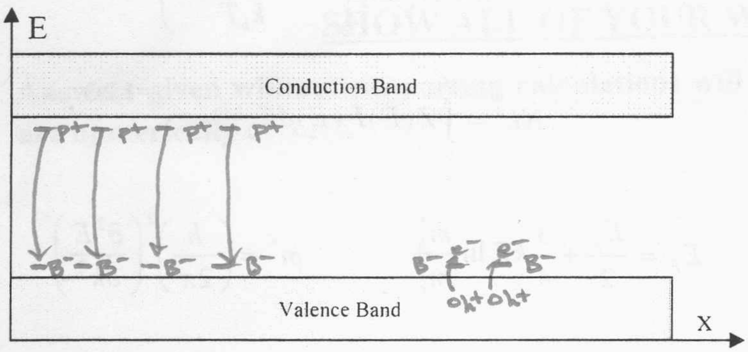
Si		GaAs	
Al:	acceptor	S_{As}:	donor
N:	none	Ge_{As}:	acceptor
Ge:	none	Ge_{Ga}:	donor
S:	double donor	Mg_{Ga}:	acceptor
Mg:	double acceptor	As_{Ga}:	double donor
Sb:	donor	Sb_{As}:	none

b) (4 pts.) Si is doped with 4×10^{17} P atoms-cm^{-3}. P is a hydrogenic impurity in Si and assume that kT is greater than the binding energy. What type is the Si? Use the diagram below to mark the P energy level and schematically show the ionization of the impurities.



n – type

c) (6 pts.) The Si from part b) is doped with an additional 6×10^{17} B atoms-cm^{-3} (so the Si is doped with P and B). Now what type is the Si? Redraw the energy level diagram, showing how both impurities ionize.



p – type

Constants

m_e=9.11x10^{-31} kg e=-1.6x10-19C h=6.625x10^{-34} J·s

1Å=1x10^{-10} m 1eV=1.6x10^{-19} J $k_b = 8.62x10^{-5} eV / K$

m_{proton}=1.67x10^{-27} kg $c = 3x10^8 m/s$ N_A=6.02x10^{23} atoms/mole

Equations:

$$\Delta p\, \Delta x \geq h \qquad \Delta E \Delta t \geq h \qquad E = \frac{p^2}{2m} = \frac{1}{2} m v^2 \qquad R = \rho \frac{L}{A}$$

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{2\pi}{k} \qquad E = \frac{h\,c}{\lambda} = h\nu \qquad E_n = \frac{n^2 h^2}{8mL^2}$$

$$\sigma = e(n\mu_e + p\mu_h) \qquad \mu = \frac{e\tau}{m} \qquad \sigma = \frac{1}{\rho}$$

$$J = NV_d e = N\mu E e \qquad E_H = R_H J B \qquad R_H = \frac{1}{eN_e}$$

time-independent Schrödinger equation:

$$\left(\frac{-\hbar^2}{2m}\right)\frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$$

common solutions:

$$\psi(x) = \left[A\exp(ikx) + B\exp(-ikx)\right] \qquad \psi(x) = C\exp(kx) + D\exp(-kx)$$

$$E = \frac{\hbar^2 k^2}{2m} \qquad E_n = \frac{-13.6}{n^2} \qquad Z(E)dE = \frac{4\pi L^3 (2m)^{3/2}}{h^3} E^{1/2} dE$$

$$n_i = \sqrt{N_c N_v}\, \exp\left(-\frac{E_g}{2k_b T}\right) \qquad F(E) = \frac{1}{1 + \exp\left[\frac{(E - E_F)}{k_b T}\right]}$$

$$p = N_v \exp\left(-\frac{E_F}{k_b T}\right) \qquad n = N_c \exp\left(-\frac{(E_g - E_F)}{k_b T}\right)$$

$$\omega = \left(\frac{4C}{M}\right)^{1/2} \left|\sin\left(\frac{ka}{2}\right)\right| \qquad NL^x = \int_0^\infty Z(E)F(E)dE \quad x=1,2,3$$

$$N_v = 2\left(\frac{2\pi m_h^* k_b T}{h^2}\right)^{3/2} \qquad E_F = \frac{E_g}{2} + \frac{3}{4} kT \ln\frac{m_h^*}{m_e^*} \qquad m^* = \left(\frac{h}{2\pi}\right)^2 \left(\frac{\partial^2 E}{\partial k^2}\right)^{-1}$$