Name (Last, First): ____________

Student ID ____________

Circle your section:

301	MWF 8-9A	121 LATIMER	LIANG
303	MWF 9-10A	121 LATIMER	SHAPIRO
306	MWF 10-11A	237 CORY	SHAPIRO
307	MWF 11-12P	736 EVANS	WORMLEIGHTON
309	MWF 4-5P	100 WHEELER	RABINOVICH
313	MWF 2-3P	115 KROEBER	LIANG
314	MWF 1-2P	110 WHEELER	WORMLEIGHTON
315	MWF 3-4P	121 LATIMER	RABINOVICH

If none of the above, please explain: ____________

Only this exam and a pen or pencil should be on your desk.

(You can get scratch paper from me if you need it.)

Problem	Points Possible	Your Score
A	10	
B	10	
C	10	
D	10	
E	10	
F	10	
G	10	
H	10	
I	10	
J	10	

Problem A. Decide if the following are **always true** or **at least sometimes false**. Enter your answers as **T** or **F** in the following chart. Correct answers receive 1 points, incorrect answers receive -1 points, and blank answers receive 0 points. No justification is necessary, although if you believe the question is ambiguous, record your interpretation below it. A is always a matrix.

Statement	1	2	3	4	5	6	7	8	9	10
Answer										

1. The equation $y''(t) = (y'(t))^2$ is a differential equation.
2. Every linear, constant coefficient, homogenous ODE has a basis of solutions of the form e^{ct} for varying c.
3. If the Wronskian of a collection of functions vanishes at any point, then the functions are linearly dependent.
4. There exists a unique solution to the equation $y''(t) + \cos(t)y'(t) + ty(t)$ with $y(0) = 1$ and $y'(0) = 2$.
5. There is a third-order, linear, constant coefficient, homogenous ODE with t^4 as a solution.
6. The motion of a spring is described by a linear ordinary differential equation.
7. The heat equation is a linear differential equation.
8. Computing Fourier coefficients can be thought of as an orthogonal projection.
9. The Fourier expansion of the function $|x|$ on the interval $[-\pi, \pi]$ has no cosine terms.
10. The space of f satisfying $\int_0^{10} f(x)dx = 0$ is a vector space.

Problem B. Give an example, or explain why none exists.

1. (3 pts) A linear partial differential equation.

2. (3 pts) A linear, constant coefficient, homogenous, second order ODE with solutions e^{2x}, e^x.

3. (4 pts) A third order linear, constant coefficient, homogenous ODE with solutions x, e^x.

Problem C.

(1 pt) Give the general solution to the equation $y' = 3y$.

(2 pts) Give the general solution to the equation $y'' + 3y' + 2y = 0$.

(3 pts) Give the general solution to the equation $y'' + 4y = 0$. Be sure to use real valued functions.

(4 pts) Give the general solution to the equation $y'' + 3y' + 2y = e^{-2t}$.

Problem D.

(10 pts) State the existence and uniqueness theorem for linear homogenous ODE. (You can use any of the various equivalent formulations.)

Problem E. Consider the equation $y''(t) - 4y(t) = 0$.

(3 pts) Find a basis for the solutions

(3 pts) Compute the Wronskian of the basis you found

(4pts) Find $y(t)$ satisfying the above equation, such that $y(0) = 1$ and $y'(0) = 1$.

Problem F. Consider the equation $y'''(t) - 2y''(t) + y'(t) - 2y(t)$.

(5pts) Find a basis of real solutions

(5pts) Solve the initial value problem $y(0) = 0$, $y'(0) = 1$, $y''(0) = 2$.

Problem G.

(2pts) Compute $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^2, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^3, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^4, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^5$

(5pts) Compute the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

(3pts) Compute $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^{100}$

Problem H.

(10 pts) Find bases for the kernel and image of the linear transformation given by the matrix:

$$M = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

Problem I.

(10pts) Consider the function $f(x)$ defined on $[0, \pi]$ by the formula

$$u(x,0) = \begin{cases} -x & x < \pi/2 \\ \pi - x & x \geq \pi/2 \end{cases}$$

Determine the sine Fourier series of this function.

Problem J.

(10 pts) Consider a wire of length π, which is stretched from $x = 0$ to $x = \pi$.

Suppose the initial temperature is given by the function

$$u(x,0) = \begin{cases} 0 & x < \pi/2 \\ \pi & x \geq \pi/2 \end{cases}$$

and that as time progresses, the ends are kept at the temperatures 0 and π respectively.

Using these initial and boundary conditions, solve the heat equation

$$\frac{\partial}{\partial t} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t)$$