Student ID ______________________

Circle your section:

301	MWF 8-9A	121 LATIMER	LIANG
303	MWF 9-10A	121 LATIMER	SHAPIRO
306	MWF 10-11A	237 CORY	SHAPIRO
307	MWF 11-12P	736 EVANS	WORMLEIGHTON
309	MWF 4-5P	100 WHEELER	RABINOVICH
313	MWF 2-3P	115 KROEBER	LIANG
314	MWF 1-2P	110 WHEELER	WORMLEIGHTON
315	MWF 3-4P	121 LATIMER	RABINOVICH

If none of the above, please explain: ______________________________

Only this exam and a pen or pencil should be on your desk.

(You can get scratch paper from me if you need it.)

Problem	Points Possible	Your Score
A	10	
B	10	
C	10	
D	10	
E	10	

Problem A. Decide if the following are **always true** or **at least sometimes false**. Enter your answers as **T** or **F** in the following chart. Correct answers receive 1 points, incorrect answers receive -1 points, and blank answers receive 0 points. No justification is necessary, although if you believe the question is ambiguous, record your interpretation below it. A is always a matrix.

Statement	1	2	3	4	5	6	7	8	9	10
Answer										

1. Any real vector space is isomorphic to $\mathbb{R}^n$ for some n.
2. Any two real vector spaces with bases of the same size are isomorphic.
3. If V is a subspace of W, then $\dim V \leq \dim W$.
4. The map $f(x) \mapsto (df/dx)^2$ is linear.
5. If X and Y are diagonalizable, then $XY = YX$.
6. All matrices have at least one complex eigenvalue.
7. The kernel of a matrix contains all nonzero eigenspaces.
8. The orthogonal complement of the orthogonal complement of the orthogonal complement of V is the orthogonal complement of V.
9. Eigenvectors with distinct eigenvalues are linearly independent.
10. For any vectors $\mathbf{v}, \mathbf{w}$, we have $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$.

Problem B. Give an example, or explain why none exists.

1. (3 pts) An orthonormal basis of $\mathbb{R}^2$ containing no standard basis vectors.

2. (3 pts) A nonzero linear map $\mathbb{P}_2 \to \mathbb{P}_2$ whose cube is zero.

3. (4 pts) A 5×5 matrix with no real eigenvalues.

Problem C. (Possibly useful fact: $\cos(\pi/3) = 1/2$ and $\sin(\pi/3) = \sqrt{3}/2$)

(3 pts) Find the eigenvalues of $\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$.

(3 pts) Find a basis in which $\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$ is diagonal.

(2 pts) Compute $\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}^{100}$. **Simplify your answer** as much as possible.

(2 pts) Describe using pictures and words the actions of $\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$ and $\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}^{100}$.

Problem D. Consider the vector space V spanned by the functions e^x, xe^x, x^2e^x.

1. (3 pts) Show that $\frac{d}{dx}$ determines a linear transformation $V \to V$.

2. (3 pts) Write the matrix of $\frac{d}{dx}$ in the basis e^x, xe^x, x^2e^x.

3. (4 pts) Is $\frac{d}{dx}$ diagonalizaable on V? Why or why not?

Problem E.

1. (3 pts) Compute all dot products (there are 6) amongst the vectors

$$(1,0,1,0),(1,0,-1-0),(0,1,0,1),(0,1,0,-1)$$

2. (3 pts) Determine the orthogonal projection of the vector $(1,3,5,7)$ to the vector space spanned by $(1,0,1,0)$ and $(1,0,-1-0)$.

3. (4 pts) Find a basis for the orthogonal complement of the line spanned by $(1,3,5,7)$.