Name (Last, First):

Student ID ________________________

Circle your section:

301	MWF 8-9A	121 LATIMER	LIANG
303	MWF 9-10A	121 LATIMER	SHAPIRO
306	MWF 10-11A	237 CORY	SHAPIRO
307	MWF 11-12P	736 EVANS	WORMLEIGHTON
309	MWF 4-5P	100 WHEELER	RABINOVICH
313	MWF 2-3P	115 KROEBER	LIANG
314	MWF 1-2P	110 WHEELER	WORMLEIGHTON
315	MWF 3-4P	121 LATIMER	RABINOVICH

If none of the above, please explain: ____

Only this exam

and a pen or pencil

should be on your desk.

(You can get scratch paper from me if you need it.)

Problem	Points Possible	Your Score
A	10	
B	10	
C	10	
D	10	
E	10	

Problem A. Decide if the following are **always true** or **at least sometimes false**. Enter your answers as **T** or **F** in the following chart. Correct answers receive 1 points, incorrect answers receive -1 points, and blank answers receive 0 points. No justification is necessary, although if you believe the question is ambiguous, record your interpretation below it. A is always a matrix.

Statement	1	2	3	4	5	6	7	8	9	10
Answer										

1. The range of a linear transformation is the column space of the corresponding matrix.
2. If the rows of the matrix of a linear transformation span, the linear transformation is onto.
3. If a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ is 1-1, then it is also onto.
4. The determinant of a sum of square matrices is the sum of the determinants.
5. The determinant of a product of square matrices is the product of the determinants.
6. For a matrix A, if A^2 is defined, then it has the same size as A.
7. If the determinant of a matrix is zero, then the matrix is invertible.
8. If A is square, then the equation $A\mathbf{x} = \mathbf{b}$ always has a unique solution.
9. If $A^2 = 0$, then $A = 0$.
10. The determinant of an integer matrix is always an integer.

Problem B. Give an example, or explain why none exists.

1. (3 pts) Three 2×2 matrices which are equal to their own square.

2. (3 pts) Two non-invertible matrices whose sum is an invertible matrix.

3. (4 pts) Two non-square matrices whose product is an invertible matrix.

Problem C.

(3 pts) Compute the product of these matrices:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 2 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & -1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 3 & 3 & 1 & -1 \\ 0 & 0 & 0 & 3 & 2 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(7 pts) Compute the determinant of the result. Explain any methods you used.

Problem D. For this problem, you should justify all your work. Consider the matrix

$$A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 1 & 0 & -1 \\ 1 & 1 & 2 & 3 \end{bmatrix}$$

1. (2 pts) Write the system of linear equations corresponding to the vector equation $A\mathbf{x} = \mathbf{b}$

2. (3 pts) Give a basis for the space spanned by the columns, chosen from among the columns; give any basis you want for the space spanned by the rows.

3. (4 pts) Give a basis for the null space.

4. (1 pt) For which $\mathbf{b}$ does the system $A\mathbf{x} = \mathbf{b}$ have a solution?

Problem E. For this question, you should explain your answers.

1. (5 pts) If T is a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$, such that

$$T\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix} \qquad T\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\2\\3\end{bmatrix} \qquad T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\3\end{bmatrix}$$

Then for which matrix A is it true that, for all vectors $\mathbf{x}$ in $\mathbb{R}^3$, $T(\mathbf{x}) = A\mathbf{x}$?

2. (5 pts) What's the volume of the parallelopiped in $\mathbb{R}^3$ whose 8 corners are

$$0, \mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{a}+\mathbf{b}, \mathbf{b}+\mathbf{c}, \mathbf{a}+\mathbf{c}, \mathbf{a}+\mathbf{b}+\mathbf{c}$$

where

$$\mathbf{a} = \begin{bmatrix}3\\1\\4\end{bmatrix}, \mathbf{b} = \begin{bmatrix}1\\5\\9\end{bmatrix}, \mathbf{c} = \begin{bmatrix}2\\6\\5\end{bmatrix}$$