Mathematics 53. Fall Semester 2018 **Professor: Daniel Tataru**

Midterm 1

Sept. 27, 2018, 8:10-9:30

Your Name: _____ (write clearly !)

Your ID: _____

TA's Name: _____

Section time: _____

Directions: This is a *closed* book exam. No calculators, cell phones, tablets, laptops and other electronic devices are allowed.
Remember: Answers without explanations will not count. You should **show your work**. Solve each problem on its own page. If you need extra space you can use the backs of the pages. The scratch paper is only for your in class use and will not be graded.

(20) 1. Consider the parametric curve in polar coordinates

$$r = \cos^2\theta, \qquad \theta \in [0, 2\pi]$$

a) Sketch the curve.
b) Compute the area enclosed by the curve.
c) Find the slope of the curve at $\theta = \dfrac{\pi}{2}$.

(20) 2. Consider the points $P = (0, 1, 1)$ and $Q = (1, 0, 1)$, and let u, v be their position vectors. Calculate/describe:

a) The triple product $u \cdot (v \times u)$.

b) The area of the parallelogram with sides u, v.

c) The parametric line L through P in the direction v.

d) The distance between the point Q and the line L.

(20) 3. Consider the parametric curve $\mathbf{r}(t) = (2t, \log t, t^2)$ for $t \in [1, 4]$.

a) Find its length.

b) Find its curvature at $t = 1$.

c) Find its unit tangent and normal vector, also at $t = 1$.

(20) 4. Let S be the surface

$$x^2 - y^2 + z^2 = 1$$

a) Identify it and sketch it.

b) Find the equation of its tangent plane at the point $(1, 2, 2)$.

(20) 5. Consider the function $f(x, y) = 2x^3 + y^2 - 6xy + 4y$.

a) Find its local maximum and minimum values and saddle points.

b) Find its global maximum and minimum inside the triangle with vertices $(0, 0)$, $(0, 6)$ and $(6, 0)$.