

Mathematics 53. Fall Semester 2018

Professor: Daniel Tataru

Midterm 2

Nov.6, 2018, 8:10-9:30

Your Name: \_\_\_\_\_ (write clearly !)

Your ID: \_\_\_\_\_

TA's Name: \_\_\_\_\_

Section time: \_\_\_\_\_

**Directions:** This is a *closed* book exam. No calculators, cell phones, tablets, laptops and other electronic devices are allowed.

**Remember:** Answers without explanations will not count. You should **show your work**. Solve each problem on its own page. If you need extra space you can use the backs of the pages. The scratch paper is only for your in class use and will not be graded. Do not use invisible pencils.

- (20) 1. In the region  $\{y > |x|\}$  introduce hyperbolic coordinates

$$y = e^u \cosh v, \quad x = e^u \sinh v$$

where the hyperbolic sine and cosine are defined by

$$\cosh v = \frac{e^v + e^{-v}}{2}, \quad \sinh v = \frac{e^v - e^{-v}}{2}.$$

- a) Compute the Jacobian  $\frac{\partial(x, y)}{\partial(u, v)}$ . [Hint:  $\cosh^2 v - \sinh^2 v = 1$ .]

- b) Sketch the image of the unit square  $\{0 \leq u, v \leq 1\}$  via this change of coordinates.

- (10) 2. Consider the curve  $C = \{\sqrt{x} + \sqrt{y} = 1\}$  starting at  $(0, 1)$  and ending at  $(1, 0)$ . Evaluate

$$\int_C y \, dx - x \, dy.$$

(30) 3. a) State Green's theorem in the region: [Sketch it first]

$$D = \{1 \leq x^2 + y^2, x^2 + 4y^2 \leq 16\}$$

b) Consider the vector field

$$F = \left( \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right).$$

Is it conservative in  $\mathbb{R}^2 \setminus \{0\}$ ? But in the first quadrant  $\{x > 0, y > 0\}$ ?

c) For the above  $F$  evaluate the integral

$$\oint_C F \cdot dr \text{ where } C = \{x^2 + 4y^2 = 16\}.$$

(15) 4. Evaluate the integral

$$\int_0^2 \int_{y-1}^1 \sqrt{x^2 + 2x + 2} \, dx \, dy.$$

- (15) 5. Let  $D$  be the square with vertices  $(0, 1)$ ,  $(1, 0)$ ,  $(0, -1)$  and  $(-1, 0)$ . Evaluate

$$\iint_D e^{x+y}(x-y)^{2018} dA.$$

- (10) 6. Consider the solid bounded by the paraboloid  $z = x^2 + y^2$  and the cone  $z^2 = x^2 + y^2$ , with density  $\rho(x, y, z) = 6z$ . Find its mass.