E120: Principles of Engineering Economics
Final Exam Solutions **May 14, 2005**
Part 1: Concepts. (15 points)

1.1 A commercial airline requires jet fuel to operate its planes. Which of the following actions would allow the company to lock in the acquisition cost of its jet fuel?

I. Buy a futures contract on jet fuel.
II. Sell a futures contract on jet fuel.
III. Buy a futures call option on jet fuel.
IV. Sell a futures call option on jet fuel.

a. I only
b. II only
c. IV only
d. I and III only
e. II and IV only

1.2 Which of the following does **Not** present an arbitrage opportunity? (Assume no transaction costs and any option can be exercised immediately.)

a. An American option sells for more than its intrinsic value prior to expiration.
b. A warrant, with exercise price of $10, sells for $1; the stock price is $13.
c. A call option, with an exercise price of $25, sells for $3; the stock price is $30.
d. A put option, with an exercise price of $10, sells for $3; the stock price is $6.
e. A convertible bond sells for less than its conversion value.

1.3 For a call and put option on the same stock, with the same exercise price E, and the same time to maturity T, the call option delta minus the put option delta is equal to _________.

a. minus one
b. zero
c. plus one
d. $N(d_1)$
e. $N(d_1)$ minus one

1.4 Given the following information: The risk-free rate is 7%, the beta of stock A is 1.2, the beta of stock B is 0.8, the expected return on stock A is 13.5%, and the expected return on stock B is 11.0%. Further, we know that stock A is fairly priced and that the betas of stocks A and B are correct. Which of the following must be true?

a. Stock B is also fairly priced.
b. The expected return on stock B is too high.
c. The expected return on stock A is too high.
d. The price of stock B is too high.
e. none of the above

1.5 Which of the following statements is **False**?

a. A project that just breaks even on an accounting basis will just barely pay back (assume the fixed assets are fully depreciated in the project life).
b. A project that just breaks even on an accounting basis will have a negative NPV.
c. A project cannot earn a positive return unless its payback period is longer than the project life.
d. If the discount rate is zero, then the project will have a discounted payback that is equal to its regular payback.
e. The relation between project life and payback period alone cannot tell you whether a project has a positive or negative NPV

Part 2: calculations. (85 points)

2. (12%) With auto loans extending 5, 6, 7 or more years these days, it is common for buyers who wish to trade in their cars after a few years to find themselves to be "upside down" on the loan, i.e. the outstanding principal on the car loan exceeds the value of the car being traded. Suppose you buy a new car for $25,000, paying nothing down. You agree to a repayment schedule of six equal annual payments beginning one year from today. The banker's required return is 10%, compounded annually. Assume the car will lose $4,000 of its value the first year and further lose $3,000 each year thereafter.

a. (4%) How much will your annual payments be?

25000 = C[1-(1/1.1)^6] /0.1;   C = 5,740.18.

b. (8%) After which loan payment will the market value of the car exceed the outstanding balance of the loan for the first time?

	Market value	*Loan balance*
Year 1:	*25000-4000=21000*	*5.740.18*[1-(1/1.1)^5] /0.1=21759.80*
Year 2:	*21000-3000=18000*	*5.740.18*[1-(1/1.1)^4] /0.1=18195.60*
Year 3:	*18000-3000=15000*	*5.740.18*[1-(1/1.1)^3] /0.1=14274.98*

After the 3rd payment, the market value of the car will exceed the outstanding balance of the loan.

Note: Another way to calculate loan balance:

*Year 1: 25000*1.1-5740.18=21759.82*

*Year 2: 21759.82*1.1-5740.18 = 18195.62*

*Year 3: 18195.62*1.1-5740.18=14275*

3. (12%) The COE firm wants to build an online document process system. The system is expected to save the pre-tax cost of $150,000 each year. The system costs $12,000 and will be depreciated as a 4-year straight line. Suppose an additional backup server is needed at the end of year 2 to enhance to system. The backup server cost $6,000 and will be depreciated over 4 years using a straight-line depreciation. The firm expects the entire system to be outdated at the end of year 4, and will sell both the initial system and the backup server at the end of year 4. The initial system has no salvage value, while the backup server has a market value of $4,000. Suppose the only increase of $16,000 of the net working capital is at the end of year 2. The required return is 10% and tax rate is 34%. Should the firm start the project?

Depreciation:

	0	*1*	*2*	*3*	*4*	*B.V.(at the end of project)*
initial		*12,000/4=3,000*	*3,000*	*3,000*	*3,000*	*0*
backup				*6,000/4=1,500*	*1,500*	*3,000*

OCF1=OCF2 = 150,000(1-0.34)+3,000(0.34) = 100,020

OCF3=OCF4 =150,000(1-0.34)+(3,000+1,500)(0.34)=100,530

	0	*1*	*2*	*3*	*4*
OCF		*100,020*	*100,020*	*100,530*	*100,530*
C.S.	*-12,000*				*0*
			-6,000		*4000-(4000-3000)(0.34)=3,660*
NWC			*-16,000*		*16,000*

NPV = -12,000+100,020/1.1+(100,020-6,000-16,000)/1.1^2 +100,530/1.1^3

+(100,530+3,660+16,000)/1.1^4

=301,027.68 > 0

Yes, the firm should start the project.

4. (15%) In class, we have shown that the equity of a firm can be viewed as a call option. The option valuation methods (e.g. replication and Black-Scholes) for stock call option are hence applicable to the equity valuation. The current value of a firm is $1,200. The firm has $1,100 in pure discount debt due in 2 year and the risk-free rate is 3% compounded continuously.

a. (8%) Assume that the firm's assets will be worth either $900 or $1,600 in 2 years. What are the current values of the firm's equity and debt?

   N=ΔS/ΔC = (1,600-900)/(500-0)=1.4

   *1.4C =1,200-900e^(-0.03*2) ; C=251.72 =>current value of equity*

   Current value of debt= 1200-251.72= 948.28

b. (7%) Now assume the standard deviation of the firm's return is 40%. What is the current value of the firm's equity?

   *C =S*N(d1)-E*e^(-R*t)*N(d2)*

   *d1= [ln(S/E)+(R+σ^2)*t]/ (σ*sqrt(t))=[ln(1,200/1,100)+(0.03+0.4^2/2)*2]/(0.4*sqrt(2))=0.542*

   *d2= d1- σ*sqrt(t) = -0.023*

   => C = 1200(0.7054)-1100*e^(-0.03*2)*(0.4920) =336.80*

5. (13%) For this question, use the following information

Maturity:	7 years
Face value:	$1,000
Conversion price:	$80
Stock price at issue:	$60
Annual coupon:	$90
YTM on similar nonconvertibles:	8%

a. (2%) What is the conversion ratio?

   1000/80=12.5

b. (2%) What was the conversion premium at issuance?

   (80-60)/60 =0.33

c. (3%) What is the straight bond value?

   Straight bond value=90[1-1/1.08^7]/0.08+1,000/1.08^7=1,052.06*

d. (3%) What is the conversion value if the current stock price is $82?

   *12.5*82 =1,025*

e. (3%) What is the minimum value of this bond if the current stock price is $82 per share?

   Max[1,052.06, 1,025] =1052.06

6. (15%)You have an investment account with Ameritrade. Assume this account prohibits you from shorting stocks, but it does allow you to perform any types of transactions on European call/put options and risk-free assets.

a. (7%) Assume you want to short Microsoft stock, show how you can create a portfolio which will give you the same reward shorting the stock. (no distribution of the stock price should be assumed)

   S+P=C+PV(E) => -S =P-C-PV(E)

   Therefore, buy one put at strike price E, sell one call at same strike price E and borrow PV(E) at risk-free rate will bring you the same payoffs as shorting the stock.

b. (8%) At this moment, the Microsoft stock is priced at $25.22, a call option to be expired in 18 months with the strike price of $24.50 is priced at $3.50, and the put option with the same expiration date and strike price is valuated at $0.83. What is the implied risk-free rate? (assume continuous compounding)

   *25.22+0.83 =3.5+24.5*e^(r*(18/12)) ; r=0.055*

7. (18%) **Parts (a) and (b) are based on the following data:** A 3-year project will cost \$60,000 to construct. This will be depreciated straight-line to zero over the 3-year life. Price per unit = \$20 and variable cost per unit = \$10. Fixed costs = \$30,000 per year. The required return = 15%. Assume no requirements for net working capital, and no salvage value of the equipments.

a. (9%) What are the cash/accounting/financial break-even point?

Cash break-even: =>OCF=0; Q=(FC+OCF)/(P-V) = 30,000/(20-10) =3,000

Accounting break-even: =>NI=0 =>OCF=D; Q=(30,000+60,000/3)/(20-10)=5,000

Financial break-even: =>NPV=0 =>-60,000+OCF[1-1/1.15^3]/0.15=0, OCF=26,278.62

Q=(30,000+26278.62)/(20-10)=5,627.86 ≈ 5,628

b. (2%) What is the DOL at the accounting break-even quantity?

DOL=1+FC/OCF=1+30,000/20,000=2.5

The following two parts are for break-even analysis in general, with straight-line depreciation.

c. (3%) Show that, when the taxes are considered, the break-even quantity can be generally calculated as $Q = \dfrac{FC + \dfrac{OCF - T \times D}{1 - T}}{P - v}$.

*OCF = [(P-v)*Q-FC](1-T)+D*T*

*=>(OCF-D*T)/(1-T) = (P-v)*Q-FC*

*=>FC+(OCF-D*T)/(1-T) =(P-v)*Q*

*=> {FC+(OCF-D*T)/(1-T)}/(P-v) =Q*

d. (4%) Show that the following two inequalities hold

Cash break-even point (Ignore taxes) >= Cash break-even point (consider taxes)

Financial break-even point (Ignore taxes) <= Financial break-even point (consider taxes)

Cash break-even point (Ignore taxes): Q_C*=(FC+0)/(P-v)*

Cash break-even point (consider taxes): Q_{CT}*={FC+(0-D*T)/(1-T)}/(P-v)*

*Since 0≥ -(D*T)/(1-T),* $Q_C \geq Q_{CT}$.

Financial break-even point (Ignore taxes): Q_F*=(FC+OCF)/(P-v)*

Financial break-even point (consider taxes): Q_{FT}*={FC+(OCF-D*T)/(1-T)}/(P-v)*

*Want to prove OCF ≤ (OCF-D*T)/(1-T), which is the same as OCF ≥D.*

Let initial capital investment = C , t =life of the project, and r=discount rate.

*Then, by straight-line depreciation, D=C/t or -C+D*t =0.*

At the same time for OCF, -C+OCF[1-(1/1+r)^t]/r = 0.

Consider the discounting effect on OCF, clearly, OCF ≥D. => $Q_F \leq Q_{FT}$.