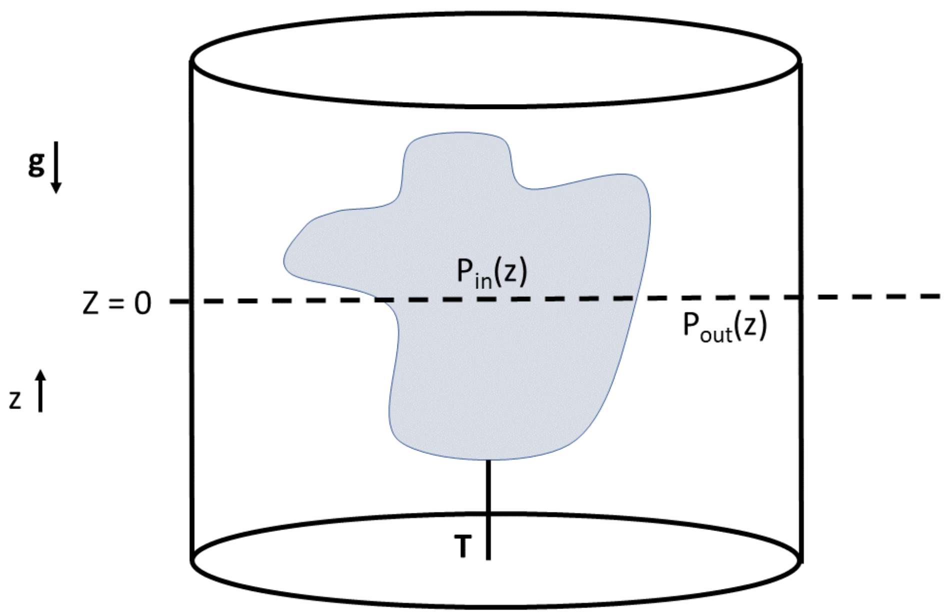
Professor Phil Marcus October 18, 2019

ME 106 Midterm Exam 1

Open notes, books, etc. No electronic devices allowed.

Consider the cylindrical container of radius R in the figure below, which is filled with a static liquid at a pressure $P_{out}(z) = a + b\,z + c\,z^2 + d\,z^3$. The bottom of the cylinder is at z=-H. Tethered to the bottom with an anchor line with tension **T** is a 3D container, shown in grey. The grey container contains a static gas within it, and this gas has pressure $P_{in}(z) = f + h\,z + k\,z^2 + n\,z^3$. The mass of the grey container (not including the gas within it) is M. The gravitational acceleration vector is $\mathbf{g} = -g\,\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector in the vertical direction.



The imaginary horizontal surface in the x-y plane at z=0, shown as a dashed line in the figure, intersects the grey container and divides it into two pieces. We indicate the region of the grey container as volume *V*. We indicate the region of the grey container that is below the dashed line as *L*. Note that the cross-sectional area of the grey container in the x-y plane at z=0 has area A_{cross}.

Note that

$$\int_V d(\text{volume}) = V_0 \qquad \int_L d(\text{volume}) = \mathrm{L}_0$$
$$\int_V z\, d(\text{volume}) = V_1 \qquad \int_L z\, d(\text{volume}) = \mathrm{L}_1$$
$$\int_V z^2\, d(\text{volume}) = V_2 \qquad \int_L z^2\, d(\text{volume}) = \mathrm{L}_2$$
$$\int_V z^3\, d(\text{volume}) = V_3 \qquad \int_L z^3\, d(\text{volume}) = \mathrm{L}_3$$

where the integrals above that are integrated over *V* are integrated over the entire volume of the grey container, and the integrals above that are integrated over *L* are integrated only over the volume of the grey container that is below the dashed line at z=0. The values of these integrals may be used in the answers to the following questions (but many of them are unnecessary):

1) Find the mass density of the fluid $\rho_{in}(z)$ inside the grey container.

2) Find the mass density of the fluid $\rho_{out}(z)$ outside the grey container, but inside the cylindrical container.

3) Find the buoyancy force on the grey container from the pressure of the fluid outside it, and also find the tension **T** in the anchor line tethering the grey container to the bottom.

4) Find the force $\mathbf{F}_{bot}$ that the pressure from the gas *inside* the container exerts on the part of the grey container boundary that is *below* the dashed line. Let's call *S* the surface on which $\mathbf{F}_{bot}$ acts. Hint: this problem is relatively easy if you choose a good control volume, so put some thought into choosing that control volume, and surround it by a fluid such that you can use the values of the integrals given above to obtain your answer. I advise using a control volume bounded by a closed surface that contains *S*. In addition, I recommend that your control volume in your thought experiment be surrounded by a fluid such that the pressure at *S* in your thought experiment is the same as the pressure at *S* in the figure above.

5) Obviously, the value of H (i.e., the depth of the bottom of the cylindrical container) and the radius of the cylinder R will not affect the value $\mathbf{F}_{bot}$. If your method of solution in problem 4 involves using the value of H or R or both, show explicitly that the values of H and R cancel so that they do not appear in your answer for $\mathbf{F}_{bot}$.