MATH 54 MIDTERM 1 – October 3 2019 5:10-6:30pm

Your Name	
Student ID	

Please exchange student IDs to record the

names of your two closest seat neighbors	

Do not turn this page until you are instructed to do so.

Show all your work in this exam booklet. There are blank pages for scratch work, but please do not remove any pages! **If you want something on an extra page to be graded, label it by the problem number and write "XTRA" on the page of the actual problem.** *In the event of an emergency or fire alarm leave your exam on your seat and meet with your GSI or professor outside.*

This exam consists of 4 problems, each of which has parts (a) and (b), in the general topic areas 1) Systems of Linear Equations, 2) Abstract Linear Algebra, 3) Linear Algebra in $\mathbb{R}^n$, 4) Matrix Algebra.

Point values are indicated in brackets to the left of each problem, add up to a total of 80, and so can be used as guide for managing the 80 minute exam time.

Each part of (a) yields full or no credit, and you don't need to show work. *To ensure credit please put each answer (and only the final answer) into the given box.*

Parts (b) can yield partial credit, in particular for explanations and documentation of your approach, even when you don't complete a calculation. In particular, if you recognize your result to be wrong (e.g. by checking!), stating this will yield partial credit. On the other hand, wrong or irrelevant statements mixed with correct work may result in reduced credit.

When asked to explain/show/prove, you should make clear and unambiguous statements that would be accessible to another student. In particular, use words or arrows to indicate how formulas relate to each other. *You may use any theorems or facts stated in the lecture notes, script, and the book sections covered by the course up to Sept.30 – after stating them clearly. If you use theorems or facts that you know from other sources, you will obtain full credit only if you include proofs that derive them from the current course material.*

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1a)

[3] $x_1 - 3x_3 = 8$
$2x_1 - x_2 + 9x_3 = 7$ is represented by the augmented matrix
$x_2 + 5x_3 = -2$

[3] A particular solution for the system represented by the augmented matrix $\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -5 & 0 & 6 \\ 0 & 0 & 0 & 1 & -7 \end{array}\right]$

is

$x_1 =$

$x_2 =$

$\vdots$

[4] The reduced echelon form of the matrix $\begin{bmatrix} 0 & 0 & 1 & 2 \\ 4 & 2 & 0 & 0 \\ 2 & 1 & 4 & 8 \end{bmatrix}$ is

[10] **1b)** Given any 2×3 augmented matrix in reduced echelon form, state conditions which guarantee that the corresponding system of 2 linear equations for 2 variables has infinitely many solutions.

Then make a list of all 2×3 augmented matrices with infinitely many solutions, using the entries 1, 0, or $\star$ (to denote entries that can be any real number).

2a)

[3] The range of a linear transformation $T : V \to W$ is the subspace consisting of

$\in$ for which .

[3] Three vectors $v_1, v_2, v_3 \in V$ are linearly independent if

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$ . .

[4] If $T : \mathbb{P}_2 \to \mathbb{P}_2$ is a linear tranformation with $T(1 + t^2) = 2t$ and $T(1 - t^2) = 6$, then

$T(t^2) =$

[10] **2b)** Determine the kernel of the linear tranformation $T : \mathbb{P}_2 \to \mathbb{R}^2$ and explain why your computation is true, *using only definitions and the following information (no theorems etc.)*:

- The equation $T(p) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ has solution set $\{p(t) = a_0 + 5t + a_2t^2 \mid a_0, a_2 \in \mathbb{R}\}$.

[3] **3a**) The linear transformation $\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x+2y \\ 4y \end{bmatrix}$ is represented by the matrix $\boxed{}$.

[4] The vectors $\begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$ are ... [check all that apply] ...

- ☐ spanning $\mathbb{R}^3$
- ☐ a basis of $\mathbb{R}^3$
- ☐ linearly dependent
- ☐ linearly independent

[3] Circle each subspace of $\mathbb{R}^2$; cross out sets that are not subspaces of $\mathbb{R}^2$.

$\{\begin{bmatrix} x \\ y \end{bmatrix} |y = 3x\}$ $\qquad$ $\{\begin{bmatrix} x \\ y \end{bmatrix} |y = 3x^2\}$ $\qquad$ $\{\begin{bmatrix} x \\ y \end{bmatrix} |y = 3x+1\}$

[10] **3b)** Find the set of solutions $\mathbf{x} \in \mathbb{R}^3$ of the equation $\begin{bmatrix} 1 & -2 & 0 \\ -2 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 10 \end{bmatrix}$.

Then state a general solution principle for inhomogeneous linear equations (no need to prove it) and explain how your result is an example of this principle.

[5] **4a)** Compute or state "not defined" for the products of $A = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$

$AB =$

$BA =$

[5] $det \begin{bmatrix} 2 & 0 & 0 & 3 \\ 3 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & 3 & 2 \end{bmatrix} =$

[10] **4b)** Compute the inverse A^{-1} of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 3 & -2 \end{bmatrix}$.

Then state a definition of "inverse matrix" and explain why it provides a general formula for solutions $\mathbf{x}$ of the equation $A\mathbf{x} = \mathbf{b}$.

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