UNIVERSITY OF CALIFORNIA

College of Engineering

Department of Mechanical Engineering and Department of Materials Science & Engineering

Fall 2008 Prof. Ritchie

MSE C 113/ME C124

Mechanical Behavior of Materials

Midterm #1 Solutions

Problem 1

Find shear stress due to Torsion

$r = 3\ m; \quad t = 30 \times 10^{-3}\ m\ ; \ p = 1000 \times 10^{3}\ Pa\ ; \ MPa = 1 \times 10^{6}\ Pa$

$$r_m = r + \frac{t}{2} = 3.015\ m$$

$$T = 10 \times 10^{6}\ N \cdot m$$

$$\sigma_{\theta z} = \frac{T}{2 \cdot \pi \cdot {r_m}^2 \cdot t} = 5.84 \times 10^{6}\ Pa = 5.84\ MPa$$

The stress due to pressure:

$$\sigma_{\theta\theta} = \frac{p \cdot r}{t} = 100.5\ MPa$$

$$\sigma_{zz} = \frac{p \cdot r}{2\ t} = 50.25\ MPa$$

a) Find the principal stresses

$$\sigma_{11} = \sigma_{\theta\theta} = 100.5\ MPa$$

$$\sigma_{22} = \sigma_{zz} = 50.25\ MPa$$

$$\sigma_{12} = \sigma_{\theta z} = 5.84\ MPa$$

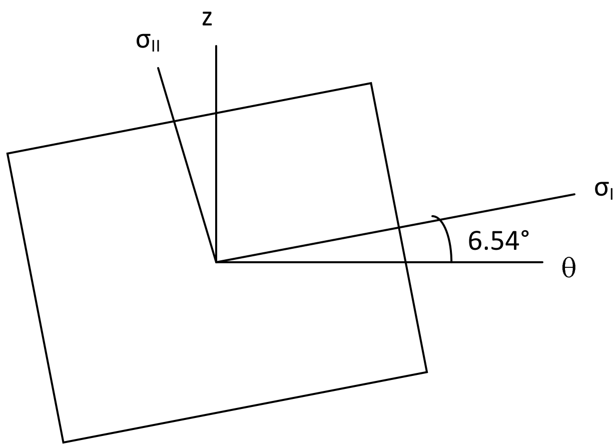
$$\sigma_I = \frac{(\sigma_{11} + \sigma_{22})}{2} + \sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + (\sigma_{12})^2} = 101.17\ MPa$$

$$\sigma_{II} = \frac{(\sigma_{11} + \sigma_{22})}{2} - \sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + (\sigma_{12})^2} = 49.58\ MPa$$

Find the principal angle

$$\tan\left(2 \cdot \theta_p\right) = \frac{2 \cdot \sigma_{12}}{\sigma_{11} - \sigma_{22}} = \frac{2 \cdot (5.35\ MPa)}{105\ MPa - 52.5\ MPa} = 6.54°$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + (\sigma_{12})^2} = 25.8\ MPa$$



b) Check for yielding

TRESCA CRITERION: $\tau = k = \sigma_y/2$

$\sigma_y = 200$ MPa ; $k = 100$ MPa → $\tau_{max} = 25.8$ MPa

$\tau_{max} < k$, therefore the cylinder does not yield.

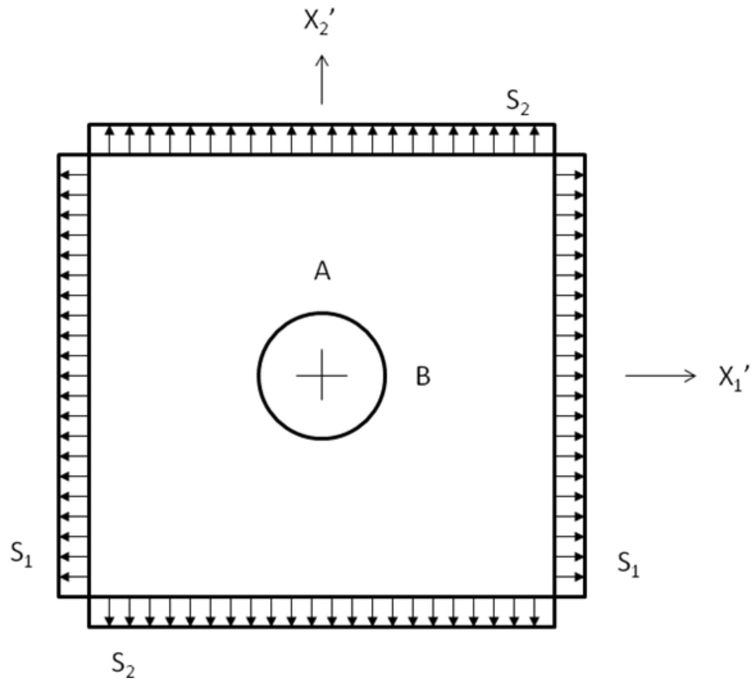
MISES CRITERION: $\sigma_{equivalent} = \sigma_y$

$$\sigma_{equivalent} = \sqrt{\frac{1}{2}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2] + 3({\sigma_{12}}^2 + {\sigma_{23}}^2 + {\sigma_{31}}^2)}$$
$$= 84.35\ MPa$$

$\sigma_{equivalent} < \sigma_y$, therefore the cylinder does not yield.

c) Hole in cylinder

The stress concentration factor and stresses on the hole can be calculated by using the principle stresses, and a stress concentration calculation.



$S_1 = \sigma_I$; $S_2 = \sigma_{II}$

$\sigma_{A\theta\theta} = 3 \cdot S_2 - S_1 = 47.57\ MPa$

$\sigma_{B\theta\theta} = 3 \cdot S_1 - S_2 = 253.93\ MPa$

At point A:

TRESCA: $\frac{\sigma_{A\theta\theta}}{2} < \frac{\sigma_y}{2}$

MISES: $\sigma < \sigma_y$ Therefore no yielding at point A

At point B:

TRESCA: $\frac{\sigma_{A\theta\theta}}{2} > \frac{\sigma_y}{2}$

MISES: $\sigma > \sigma_y$ Therefore yielding at point B

<u>Yielding does occur at the edge of the hole.</u>

Problem 2

<u>Load to yield steel:</u>

$\sigma_{11} = 1240\ MPa$

A = 1 m^2

$P_{Steel} = \sigma_{11} * A = 1.24 \times 10^9$ N

<u>Load to yield silver:</u>

Triaxial stresses develop due to constraint.

$\sigma_{11} = P/A$

$$\epsilon_{11,Ag} = \frac{\sigma_{11} - \upsilon_{Ag}(\sigma_{22} + \sigma_{33})}{E_{Ag}}$$

$$\epsilon_{22,Ag} = \frac{\sigma_{22} - \upsilon_{Ag}(\sigma_{11} + \sigma_{33})}{E_{Ag}}$$

$$\epsilon_{33,Ag} = \frac{\sigma_{33} - \upsilon_{Ag}(\sigma_{11} + \sigma_{22})}{E_{Ag}}$$

Strains in Silver in 22 and 33 directions are controlled by the steel. In the 11 direction, the strains are different.

For Steel, since there are no constraints, the only stress affecting it is the applied stress

$$\epsilon_{22,Steel} = \frac{-\upsilon_{Steel} \cdot \sigma_{11}}{E_{Steel}}$$

$$\epsilon_{33,Steel} = \frac{-\upsilon_{Steel} \cdot \sigma_{11}}{E_{Steel}}$$

$$\epsilon_{22,Steel} = \epsilon_{22,Ag}$$

$$\epsilon_{33,Steel} = \epsilon_{33,Ag}$$

Thus,

$$\frac{-\upsilon_{Steel} \cdot \sigma_{11}}{E_{Steel}} = \frac{\sigma_{22} - \upsilon_{Ag}(\sigma_{11} + \sigma_{33})}{E_{Ag}}$$

$$\frac{-\upsilon_{Steel}\cdot\sigma_{11}}{E_{Steel}} = \frac{\sigma_{33}-\upsilon_{Ag}(\sigma_{11}+\sigma_{22})}{E_{Ag}}$$

$\sigma_{22} = \sigma_{33}$

Which then simplifies to

$$\sigma_{22} = \sigma_{33} = \frac{\sigma_{11}\cdot\left(\upsilon_{Ag}-\upsilon_{Steel}\cdot\frac{E_{Ag}}{E_{Steel}}\right)}{1-\upsilon_{Ag}}$$

To find P use the MISES criterion and the known yield strength:

$$\sigma_{y,Ag} = \sqrt{\frac{1}{2}[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2]}$$

$$\sigma_{y,Ag} = \frac{1}{\sqrt{2}}\cdot\sqrt{\left[\left(\sigma_{11}-\frac{\sigma_{11}\cdot\left(\upsilon_{Ag}-\upsilon_{Steel}\cdot\frac{E_{Ag}}{E_{Steel}}\right)}{1-\upsilon_{Ag}}\right)^2+\left(\frac{\sigma_{11}\cdot\left(\upsilon_{Ag}-\upsilon_{Steel}\cdot\frac{E_{Ag}}{E_{Steel}}\right)}{1-\upsilon_{Ag}}-\sigma_{11}\right)^2\right]}$$

$$\sigma_{y,Ag} = \frac{1}{\sqrt{2}}\cdot\sqrt{2\cdot\sigma_{11}{}^2\left[1-\frac{\left(\upsilon_{Ag}-\upsilon_{Steel}\cdot\frac{E_{Ag}}{E_{Steel}}\right)}{1-\upsilon_{Ag}}\right]^2}$$

$$\sigma_{11} = \frac{\sigma_{y,Ag}}{\left[1-\frac{\left(\upsilon_{Ag}-\upsilon_{Steel}\cdot\frac{E_{Ag}}{E_{Steel}}\right)}{1-\upsilon_{Ag}}\right]} = \frac{140\ MPa}{\left[1-\frac{\left(0.367-0.3\cdot\frac{30.3\ GPa}{200\ GPa}\right)}{1-0.367}\right]} = 284.54\ MPa$$

$P_{Ag} = \sigma_{11} * A = 2.85 \times 10^8$ Newtons

<u>The silver braze yields before the steel.</u>

Problem 3

P = 800 x 10^3 Pa ; r = 3 m ; t = 10 x 10^{-3} m

r/t >10 , so the thin-wall approximation holds

a) From homework 1, we derived the following:

$$\sigma_{rr} \approx 0$$

$$\sigma_{\theta\theta} = \frac{P\,r}{t} = 240\ MPa$$

$$\sigma_{zz} = \frac{P\,r}{2\,t} = 120\ MPa$$

b) τ_{max} = radius of Mohr's circle = R

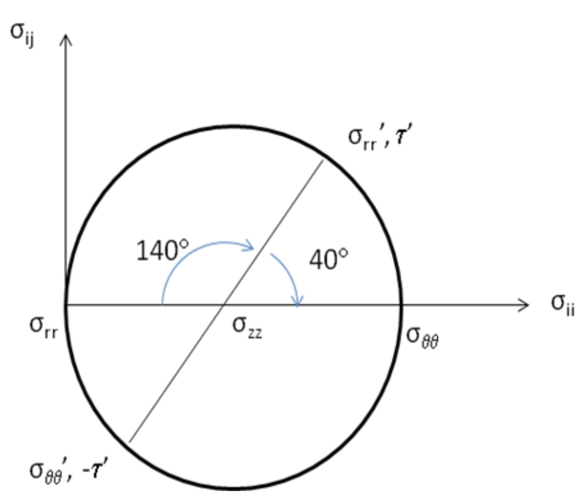
For a pressurized cylinder, σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} are your principal stresses, so

$\tau_{max} = (\sigma_{\theta\theta} - \sigma_{rr})/2$

$\underline{\tau_{max} = 120\text{ MPa}}$

Plot $\sigma_{\theta\theta}$, σ_{rr}, and σ_{zz} on another Mohr's circle to find the stresses on the weld.

Again, since $\sigma_{\theta\theta}$ and σ_{rr} are max and min principal stresses, R = $(\sigma_{\theta\theta} - \sigma_{rr})/2$ = 120 MPa and the center is at 240 – 120 = 120 MPa



$$\sigma'_{\theta\theta} = 120 - 120\cos(40°) = 28.07\ MPa$$

$$\sigma'_{rr} = 120 + 120\cos(40°) = 211.93\ MPa$$

$$\sigma'_{zz} = 120\ MPa$$

$$\tau' = 120\sin(40°) = 77.13\ MPa$$

c) Internal Pressure – a lot of people did not consider σ_{rr}. Technically, you have to consider it, because it tells you that the vessel yields at half the pressure you would calculate if you did not consider it. However, some students came up to me during the exam and asked whether they should consider it, and I told them no. Therefore, we did not take off points for anyone who made that mistake. We will in the future, though.

TRESCA:

$\tau_{max} = \tau_y$ $\quad\quad$ $\tau_y = \sigma_y /2 = 250$ MPa

From above, $\tau_{max} = (\sigma_{\theta\theta} - \sigma_{rr})/2 = (P\,r)/(2\,t)$
$P_y = 2\,t\,\tau_y / r$

<u>$P_y = 1.67$ MPa</u>

MISES

$$\sigma_{equivalent} = \sigma_y = \sqrt{\frac{1}{2}[(\sigma_{\theta\theta} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{rr})^2 + (\sigma_{rr} - \sigma_{\theta\theta})^2] + 3({\sigma_{r\theta}}^2 + {\sigma_{\theta z}}^2 + {\sigma_{rz}}^2)}$$

$\sigma_{rr} = \sigma_{r\theta} = \sigma_{zr} = \sigma_{\theta z} = 0$

and $\sigma_{\theta\theta} = 2\,\sigma_{zz} = Pr/2t$

$$\sigma_{equivalent} = \sqrt{\frac{1}{2}[(2\,\sigma_{zz} - \sigma_{zz})^2 + (\sigma_{zz})^2 + (-2\,\sigma_{zz})^2]}$$

$$\sigma_{equivalent} = \sqrt{\frac{1}{2}[6 \cdot {\sigma_{zz}}^2]} = \sqrt{\frac{1}{2}\left[6 \cdot (P\frac{r}{2t})^2\right]}$$

$P_y^2 = (4\,\sigma_y^{\,2}\,t^2)/(3\,r^2)$

<u>$P_y = 1.92$ MPa</u>