UNIVERSITY OF CALIFORNIA AT BERKELEY

Physics 7C – (Aganagic)

Fall 2019

FIRST MIDTERM

Please do all your work **in a blue/green book**. Write your name, SID, discussion section number, and GSI's name.

You are responsible for making all of your reasoning clear. **Remember to circle your final answer!**

Problem 1 (30 points)

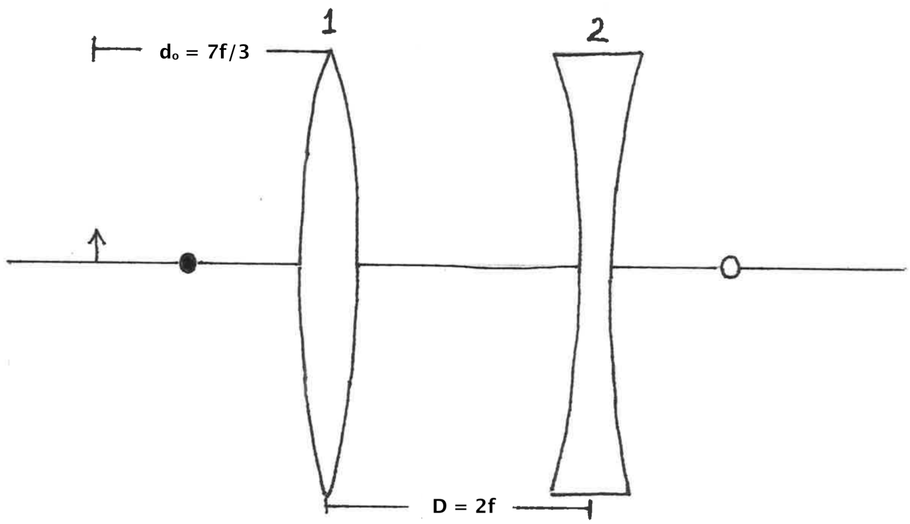
You are given a slightly flexible spherical mirror that can bend to a range of radii of curvature $0.8\text{m} \leq R \leq 10\text{m}$. You stand with your back to a giant sequoia tree which has a height of 80m.

(a) If the tree is 19.5m behind you and you hold the mirror out at a relaxed arm's length ($x = 50$ cm), can you focus the image of the tree on your eye by bending the mirror to a specific curvature? If so, how tall is the image of the tree? If not, how much further would you need to bend the mirror to focus the image?

(b) Next you allow the mirror to relax to its maximum radius of curvature and give it to your friend to hold 8m away from you. How close to the tree should you stand so that the image of the tree is again focused on your eye?

(c) Last, you bring the mirror closer so that it is now 10cm from your face. If you want the image of the tree to be 8cm tall, where should you stand and how far should you bend the mirror? Is this possible, or will the mirror break?

Problem 2 (30 points)



Two thin lenses, one converging and one diverging, are aligned horizontally. The two foci of the converging lens (number 1) are a distance f from its center; one such focus is shown by the filled circle. Similarly, the two foci of the diverging lens (number 2) are also a distance f from its center; the open circle shows one of these foci. The lens centers are a distance $D = 2f$ apart.

A small, upright object (arrow in the sketch) is placed a distance $d_o^1 = 7f/3$ in front of lens 1. If lens 2 were absent, lens 1 would form an image of this object.

(a) Find d_i^1, the distance of the image from lens 1. Is this image virtual or real?

With lens 2 in place, the image you found in (a) does not actually form. However, the combined lenses do form an image.

(b) Find d_i^2, the distance of this image from lens 2. Is this image virtual or real?

(c) Carefully draw a ray diagram showing how the image you found in (b) is formed. You need not show in detail the formation of the *first* image; just place it in your drawing, using the location and orientation you found in (a).

Problem 3 (40 points)

Consider an electromagnetic wave with electric field

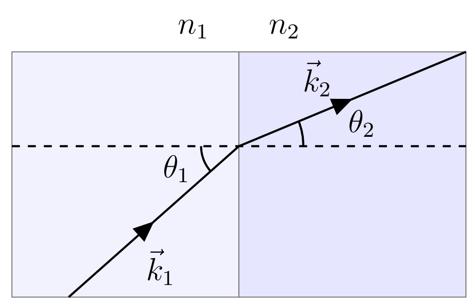
$$\mathbf{E}(y,t) = (ky + \omega t)^4 \hat{\mathbf{x}}, \tag{1}$$

where $k, \omega > 0$.

(a) Show that this satisfies the wave equation. What is the speed of the wave propagation? Suppose this wave travels in a material of index of refraction n_1. How is the speed of the wave propagation related to the speed of light c in vacuum?

(b) What is the direction of propagation of this wave? Along what axis does the corresponding magnetic field oscillate?

Now suppose that this wave enters a new material with index of refraction n_2, as in the figure below. Indicate what x, y and z directions from (1) may correspond to in the figure. (This coordinate system is adapted to the wave in the first material, and is not so great for the second. Do not worry about that, it will not be relevant for the rest.)



(c) As the electromagnetic wave transitions from the first material to the second, the frequency of the wave must stay constant. Use this to show that $k_1/k_2 = n_1/n_2$, where k_1 is the wavenumber of the wave in the first material (and similarly for k_2).

(d) The wave vector $\vec{k}$ is a vector of magnitude k, in the direction of wave propagation. As any vector, $\vec{k}$ can be decomposed as $\vec{k} = \vec{k}_{||} + \vec{k}_{\perp}$ into a component $\vec{k}_{\perp}$ along the normal to the surface, and component $\vec{k}_{||}$, which is parallel to it. Like the frequency, it turns out that $\vec{k}_{||}$ must be the same on both sides of the surface. Use this fact, and the result from (c) to derive Snell's law. (The condition $\vec{k}_{||}$ is the same on both sides of the surface is the same as asking for wave-fronts, *i.e.* surfaces of constant phase, to match up.)