Orthopaedic Biomechanics
Fall 2019 Midterm Exam

Name: SOLUTIONS

Please write your name at the top of each page as indicated. Write all answers in the space provided. If you need additional space, write on the back side of the page and clearly mark which problem you are working on. Do not remove any pages. *Good luck!*

1. Statics Analysis [25 points total]

A person is momentarily at rest in a horizontal position at the bottom of a pushup exercise on a horizontal flat surface (Figure 1). Assume the following:

- both hands and feet are on the ground
- ground reaction forces F_1 and F_2 are vertical, as shown (assume no friction)
- total body weight **W** acts vertically through point **CM**
- joint contact force at the elbow acts at white point (at distance d_3 from CM)
- ~~d_1 = 1.8 m~~, ~~d_2 = 1.0 m~~, d_3 = 0.2 m

$d_1 = 1.0m$ $d_2 = 1.8m$

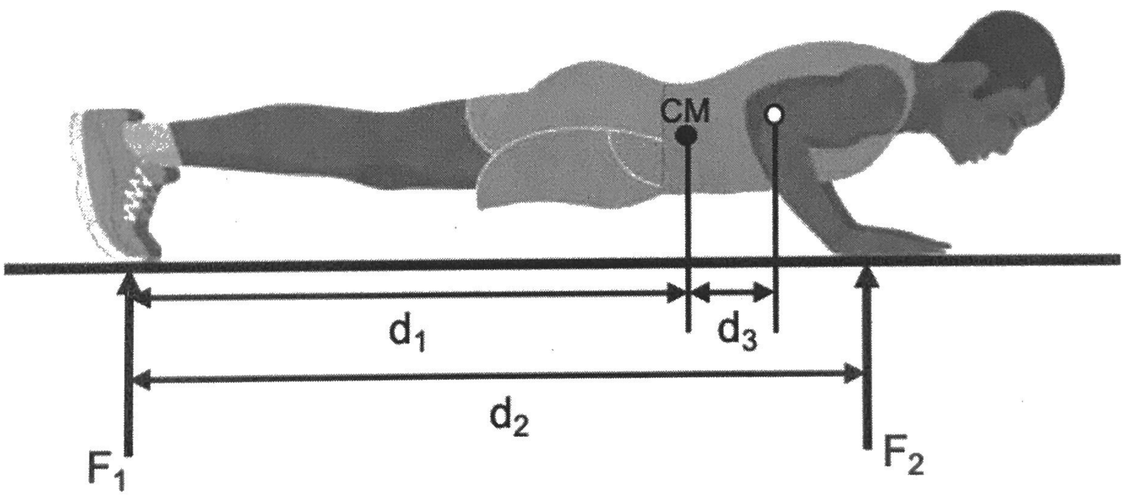
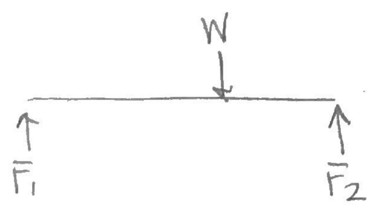


Figure 1: Push-up

a. **[5 points]** For this position, what is the magnitude of the ground reaction force acting on **each hand** in terms of body weight **W**?



$\Sigma M_{F_1} = 0 = -W(d_1) + F_2 d_2$

$W(1) = F_2(1.8)$

$F_2 = 0.56W$

↳ each hand = $\boxed{0.28W}$

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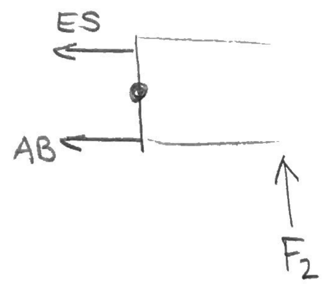
b. **[5 points]** Consider a vertical cross-section going through CM. If only one muscle is dominant at that cross-section, would that muscle be the erector spinae or the abdominal muscle? Briefly explain.



- F_2 creates a (+) ccw moment about joint center
- In order to balance this moment ($\Sigma M=0$), ABDOMINAL muscles must be dominant (- cw moment)

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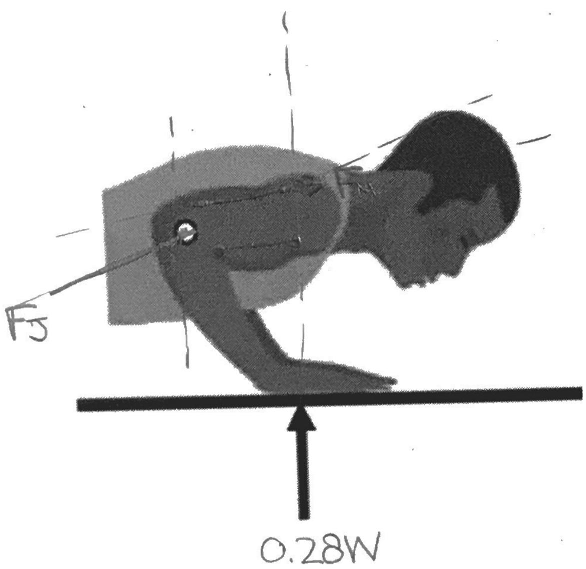
c. **[15 points]** Consider the elbow joint. Using the picture below and a **graphical/vector force triangle approach**, estimate both the direction and magnitude (in terms of body weight **W**) of the joint contact force at the elbow. Hint: assume just one muscle is dominant and ignore the mass of the forearm/hand.



which muscle is dominant?

→ 0.28 W provides a ⊕ CCW moment about the elbow joint

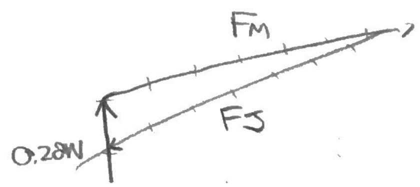
→ Triceps must be dominant (to create a ⊖ CW moment)

joint contact force @ elbow

$6.5(0.28W) = F_M$

$7(0.28W) = F_J$

$F_J = 1.96\,W$



Note: Answer is approximate. Looking for correct approach rather than exact answer

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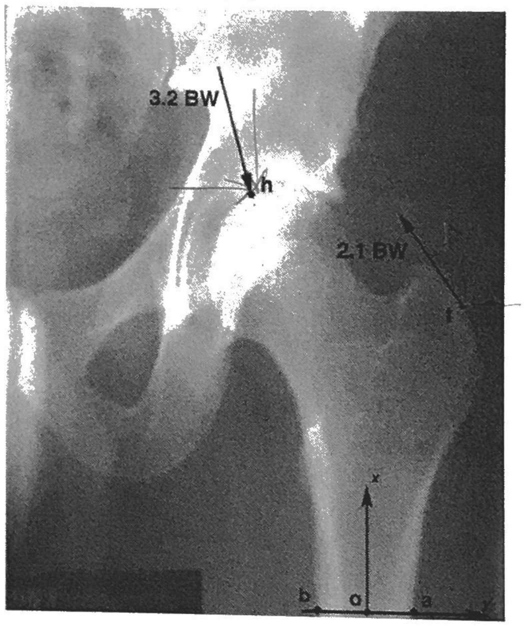
2. More Statics [20 points total]

Assume during the single-legged stance phase of walking that the hip joint contact force and the abductor forces at the proximal femur are as shown below, and no other muscle or ligament forces are acting across the hip joint.



a. **[5 points]** Ignoring any inertial effects at this instant, what is the orientation (clockwise or counter-clockwise) of the moment acting on the bone about point **o**? <u>Explain your answer</u> (no points without an explanation!)

2.1BW → + CCW moment about O

3.2BW → goes through O ∴ generates no moment

<u>Note:</u>

In order to balance the + CCW moment generated by 2.1 BW at t, the reaction moment in the bone at O must be − CW

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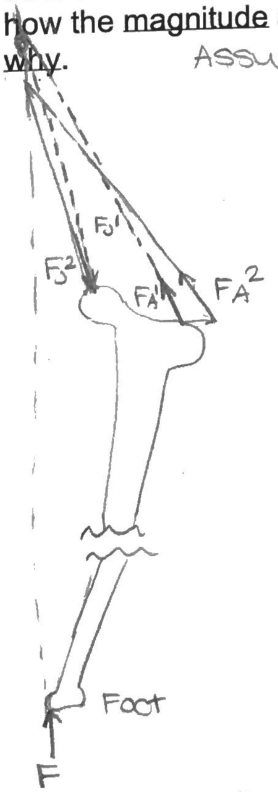
b. **[5 points]** Cortical bone tissue is known to be stronger in compression than tension. How does that attribute relate to the internal loading of the bone at the mid-diaphysis?

- Due to bending
  - ↳ Tissue at b will be in compression
  - ↳ Tissue at a in tension
- Entire bone under axial compression
- Since the axial and bending ~~stresses~~ are additive, the magnitude of the stress at b (compressive) will be larger than that at a.

c. **[10 points]** Imagine now that an orthopaedic surgeon surgically moved the greater trochanter laterally, so that point ***t*** was shifted a little to the right. Describe qualitatively how the magnitude and direction of the joint contact force would change, and explain why.

Assume: – Weight of leg is negligible
– Abductor muscles dominate
– Acetabular attachment remains unchanged post-surgery

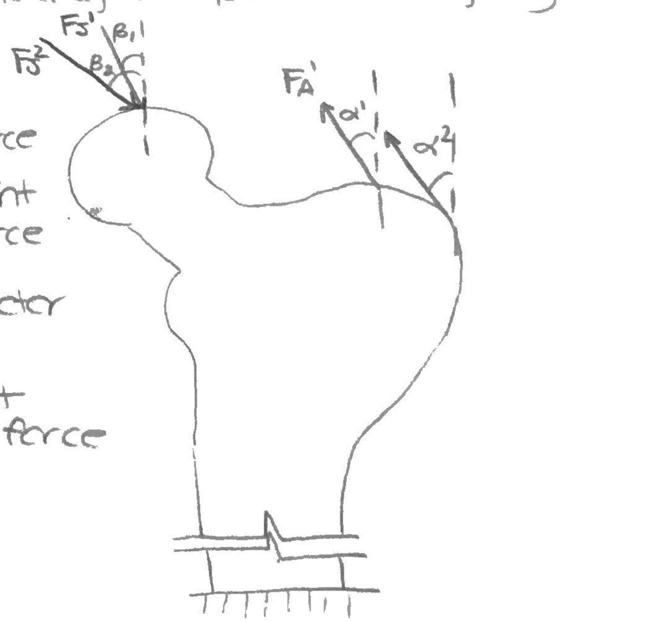


F_A' = original abductor force

F_J' = original joint contact force

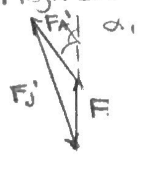
F_A^2 = new abductor force

F_J^2 = new joint contact force

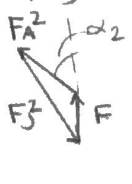


→ using vector force triangle

original



post-surgery



→ Magnitude of F_J (joint contact force) reduced

→ Angle from vertical increases

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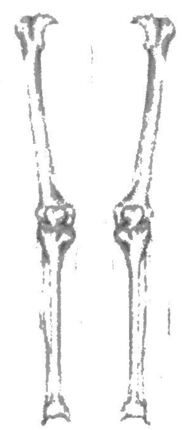
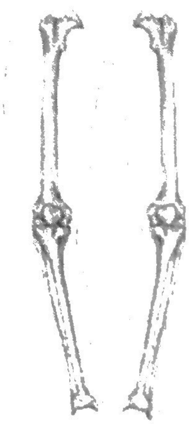
3. Joint Stability [15 points total]
In a normal knee joint, when you are standing at rest, the center of the knee joint is typically directly over the center of the ankle joint, so that the ground reaction force at the foot goes through the center of the knee joint. By contrast, in a bow-legged individual ("varus" alignment), the knee joint is typically displaced laterally with respect to the foot, as shown below.

For these two situations, compare the magnitude of the contact force on the medial tibial condyle of the knee joint (normal vs. varus situations). For both situations assume that the total ground reaction force: 1) equals body weight; 2) is the same for both feet; 3) is purely vertical; and 4) goes through the center of the ankle joint. In answering this question, do the following:

- draw a free-body diagram of each situation 5/5
- perform an (analytical) equilibrium analysis for each situation; add in your diagram any relevant dimensions and clearly label them 3
- estimate the ratio of the contact force on the medial tibial condyle for varus with respect to normal (i.e. varus value / normal value). 2



Normal Alignment



Varus Alignment

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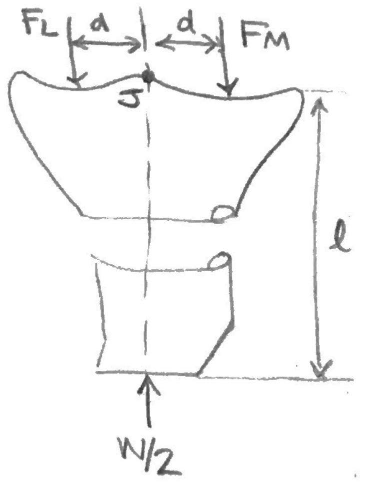
<u>Assume</u>

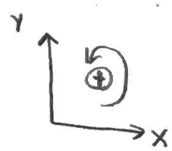
↳ knee geometry is symmetrical

↳ Ground reaction force acts at the same place on the foot for each case

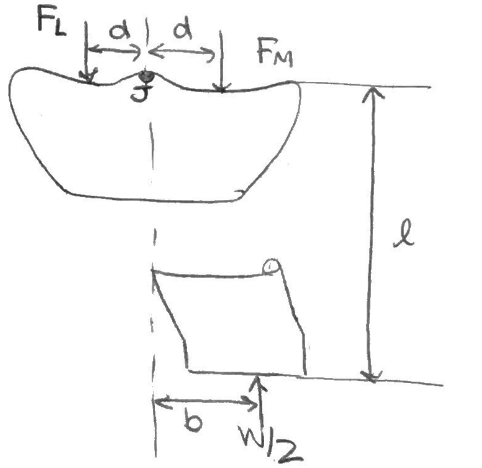
* consider the forces acting on the <u>right</u> tibia

<u>FBD of right leg, Normal</u>





<u>FBD of right leg, VARUS</u>



$\Sigma M_J = 0 = F_L d - F_M d$

$F_L = F_M$

$\Sigma F_y = 0 = \frac{W}{2} - F_L - F_M$

$F_L = F_M = \frac{W}{4}$ ← normal ⟶ ratio

$\Sigma M_J = 0 = F_L d - F_M d + (W/2) b$

varus ⟶ $F_M = F_L + \frac{Wb}{2d}$

$F_M > F_L \Rightarrow F_M > \frac{W}{4}$

ratio $\frac{\text{varus}}{\text{normal}} = \frac{F_L + \frac{Wb}{2d}}{W/4}$

- Ratio of F_M for varus with respect to normal will be greater than 1
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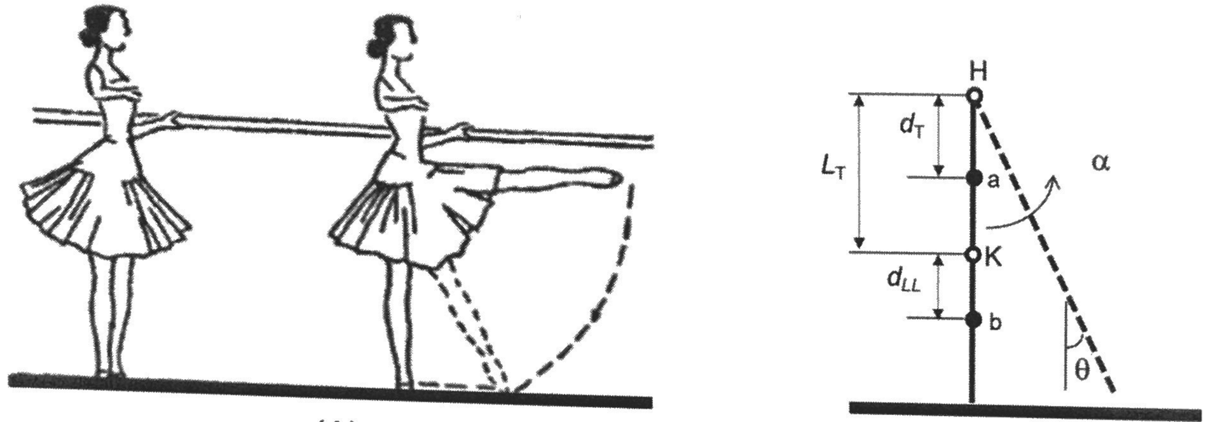
4. Dynamic Analysis [40 points total]

A ballerina executes a grand battement, a movement in which both legs are kept straight, and one leg is kicked outward from the body in the sagittal plane (hip flexion). The following question pertains to the flexed leg.



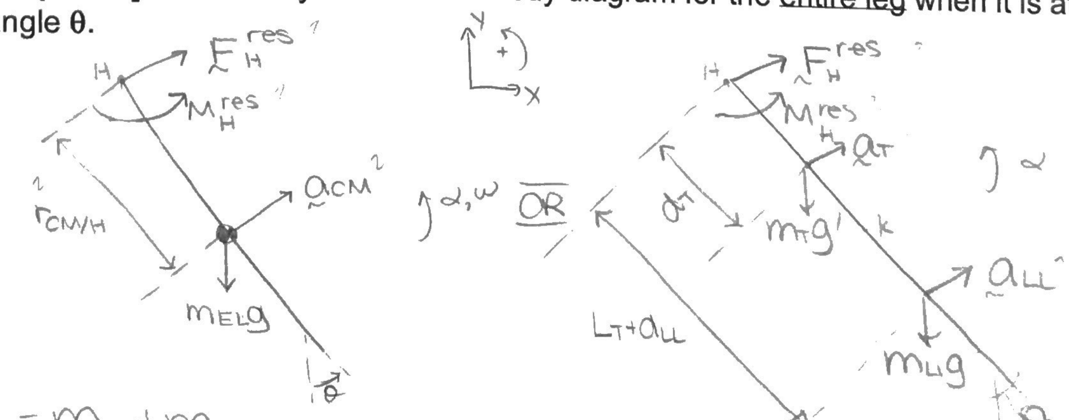
(A) (B)

Figure 2: Ballerina executing a grand battement, where the right leg is flexed. (B) Schematic of the flexed leg in its initial position (solid line) and the movement of the leg about the hip during hip flexion (dashed line).

For this two-dimensional planar rigid body problem assume the following:

- The hip joint is at point ***H***, a fixed point in space.
- The entire leg consists of the thigh (the leg above the knee joint **K**) and the lower leg (the leg below the knee joint, including the foot, with a rigid joint at the ankle).
- The observed angular acceleration for the leg when it is at the angle θ is α.
- The center of mass of the thigh (**m_T**) is at point ***a***, located a distance **d_T** from the hip joint ***H***.
- The center of mass of the lower leg and foot (**m_{LL}**) is at point ***b***, located a distance **d_{LL}** from the knee joint ***K***.
- The length of the thigh is **L_T**.
- The mass of the entire leg is **m_{EL}**, and is equal to the sum of the thigh and the lower leg and foot ($m_{EL} = m_T + m_{LL}$)
- **I_{EL}** is the mass moment of inertia of the entire leg about its center of mass.
- **I_{LL}** is the mass moment of inertia of the lower leg about its center of mass.

a. **[10 points]** Draw a fully labeled free body diagram for the entire leg when it is at angle θ.



$$m_{EL} = m_T + m_{LL}$$

$$r_{CM/H} = \frac{d_T m_T + (L_T + d_{LL}) m_{LL}}{m_T + m_{LL}}$$

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b. **[15 points]** Write out an equation for the *resultant moment at the hip joint*, expressed in terms *of only* **θ, α,** and the provided geometry and mass properties.

- H is fixed $\Longrightarrow \sum M_H = I_H \alpha$

RHS $\longrightarrow$ $\left\{ \begin{array}{l} I_H = I_{EL} + (r_{CM/H})^2 (m_{EL}) \\ \text{where } r_{CM/H} = \dfrac{d_T m_T + (L_T + d_L) m_L}{m_{EL}} \end{array} \right.$

LHS $\quad -m_{EL}\, g\, r_{CM/H} \sin\theta + M_H^{Res}$

- Putting it together...

$$-m_{EL}\, g\, r_{CM/H} \sin\theta + M_H^{Res} = \left[I_{EL} + (r_{CM/H})^2 m_{EL} \right] \alpha$$

- solving for M_H^{res}...

$$M_H^{res} = \left[I_{EL} + \left(\frac{d_T m_T + (L_T + d_L) m_L}{m_{EL}} \right)^2 (m_{EL}) \right] \alpha + m_{EL}\, g \left[\frac{d_T m_T + (L_T + d_L) m_L}{m_{EL}} \right] \sin\theta$$

- simplify...

$$\boxed{M_H^{res} = \left[I_{EL} + \frac{(d_T m_T + (L_T + d_L) m_L)^2}{m_{EL}} \right] \alpha + \left[d_T m_T + (L_T + d_L) m_L \right] g \sin\theta}$$

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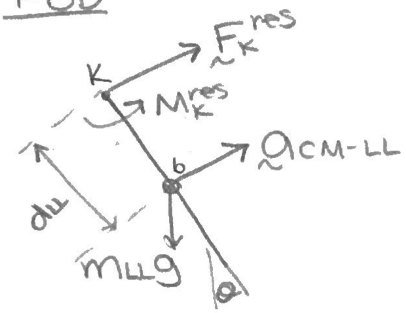
c. **[15 points]** Write out an equation for the *resultant moment at the knee joint* required to keep the leg straight, expressed in terms *of only* **θ, α,** and the provided geometry and mass properties.

- K is accelerating $\Rightarrow \sum M_K = I_{CM}\alpha + r_{CM/K} \times m\underset{\sim}{a}_{CM}$

$$\sum M_K = I_{LL}\alpha + \underbrace{d_{LL} \times m_{LL}\underset{\sim}{a}_{CM\text{-}LL}}_{(i)}$$

5 FBD



(i) $d_{LL} \times m_{LL}\underset{\sim}{a}_{CM\text{-}LL} = d_{LL} m_{LL} \underset{\sim}{a}_{T\text{-}LL}$

↳ $a_{CM\text{-}LL} = \underset{\sim}{a}_{T\text{-}LL} + \underset{\sim}{a}_{R\text{-}LL}$

↳ since $a_{R\text{-}LL}$ is collinear with d_{LL}, this term drops ($d_{LL} \times m_{LL} a_{R\text{-}LL} = 0$)

↳ $a_{T\text{-}LL} = r_{H/b}\,\alpha$ (rotating about H)

$r_{H/b} \downarrow L_T + d_{LL}$

$= d_{LL} m_{LL} (L_T + d_{LL})\alpha$

RHS $I_{LL}\alpha + d_{LL} m_{LL} (L_T + d_{LL})\alpha$

LHS $M_K^{res} - m_{LL} g d_{LL} \sin\theta$

$$\boxed{M_K^{res} = I_{LL}\alpha + d_{LL} m_{LL} (L_T + d_{LL})\alpha + m_{LL} g d_{LL} \sin\theta}$$