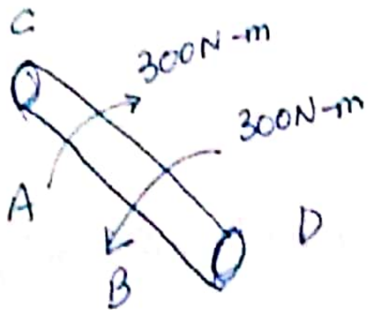
1.

a)



$$M_A = \underset{kN}{(10-4)} \cdot r_A = 6 \times \cancel{50}\ 100 = \underline{\cancel{300}\ 600 \text{ N-m}}.$$

$$M_B = \underset{kN}{(6-2)} \cdot r_B = 4 \times \cancel{75}\ 150 = \underline{\cancel{300}\ 600 \text{ Nm}}$$

$\therefore$ In the region A-B, T is constant and is equal to $\underline{\cancel{300}\ 600 \text{ N-m}}$.

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\therefore \theta_{A-B} = \frac{TL}{GJ} = \frac{\cancel{300}\ 600 \times 0.45}{81 \times 10^9 \times \frac{\pi}{2} \times \left(\frac{10}{3} \times 10^{-2}\right)^4}$$

$$= \frac{\cancel{600}\ 300 \times 0.45}{81 \times 10^9 \times \pi \times \frac{10^{-4}}{81}}$$

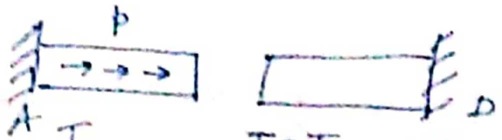
$$= \frac{\cancel{270}\ 540}{\pi} \times 10^{-5} \text{ rad.}$$

$$= \underline{\frac{\cancel{27}\ 54}{\pi} \times 10^{-4} \text{ rad.}}$$

b) $\frac{T}{J} = \frac{G\theta}{L} = \frac{2T}{\pi R^4}$

If we apply a constant twisting moment T over a circular shaft with radius R and length L, and measure the angle of twist θ between its ends, we can find G.

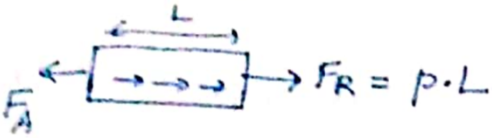
2.



$T_0 \qquad T = T_0 + \Delta T$

a) $\underline{T < T_1}$.

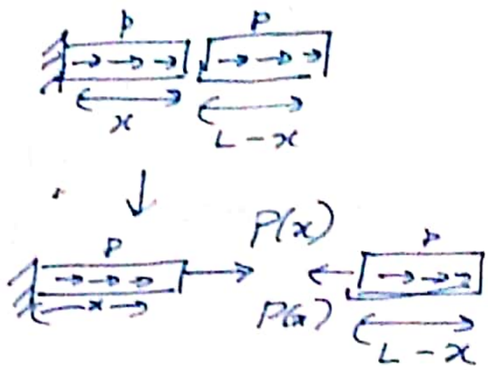
$\sigma_A = \dfrac{p \cdot L}{A}$. $\qquad \sigma_D = 0$ (no constraint yet)



$F_A = p \cdot L$

b) $\delta_{therm} + \delta_{mech} = g_0$.

$\delta_{therm} = \alpha \Delta T_1 \cdot L$



$$\delta_{mech} = \int_{x=0}^{x=L} \frac{P(x)\,dx}{AE}$$

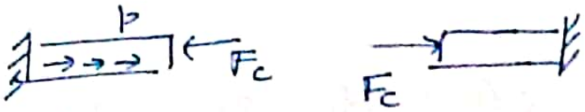
$$= \int_0^L \frac{p \cdot (L-x)}{AE}$$

$P(x) = p \cdot (L - x)$ from right side.

$$= \frac{p}{AE}\left(Lx - \frac{x^2}{2}\right)\Bigg|_0^L = \underline{\frac{pL^2}{2AE}}.$$

$$\therefore \alpha \Delta T_1 \cdot L + \frac{pL^2}{2AE} = g_0$$

$$\therefore \Delta T_1 = \underline{\frac{\left(g_0 - \dfrac{pL^2}{2AE}\right)}{\alpha L}}.$$

c) 

An additional F_c will halt free expansion of right rod.

$$\delta_{right} = \alpha \Delta T_2 \cdot L - \frac{F_c \cdot L}{AE}$$

$$\delta_{left} = \int_0^L \frac{P(x)\,dx}{AE} = \int_0^L \frac{(p(L-x) - F_c)\,dx}{AE}$$

$$= \frac{pL^2}{2AE} - \frac{F_c \cdot L}{AE}$$

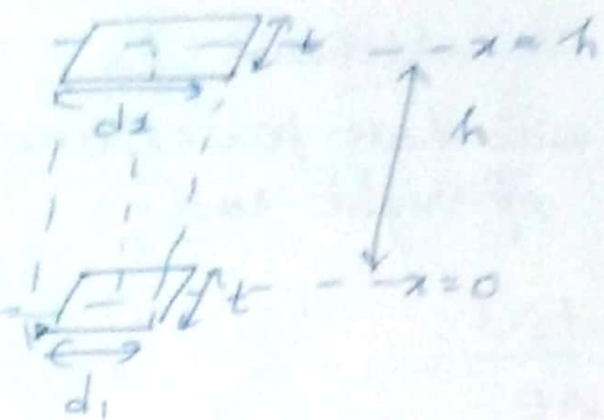
$$\delta_{left} + \delta_{right} = g_0$$

$$\Rightarrow \alpha \Delta T_2 \cdot L + \frac{pL^2}{2AE} - \frac{2F_c \cdot L}{AE} = g_0$$

$$\Rightarrow F_c = \left(\alpha \Delta T_2 L + \frac{pL^2}{2AE} - g_0\right) \cdot \frac{AE}{2L}$$

Scanned by CamScanner

3.



$$d(x) = d_1 + \left(\frac{d_2 - d_1}{h}\right) \cdot x$$

$$A(x) = \left\{ d_1 + \left(\frac{d_2 - d_1}{h}\right) x \right\} \cdot t$$

$$\therefore \delta = \int \frac{P(x)\,dx}{A(x)\,E(x)}$$

$$= \frac{P}{Et} \int_0^h \frac{dx}{\left(d_1 + \left(\frac{d_2 - d_1}{h}\right) x\right)}$$

$$= \frac{P}{Et} \cdot \frac{1}{\left(\frac{d_2 - d_1}{h}\right)} \int_0^h \frac{dx}{x + \frac{d_1 h}{d_2 - d_1}}$$

$$= \frac{Ph}{Et(d_2 - d_1)} \ln \left| x + \frac{d_1 h}{d_2 - d_1} \right| \Bigg|_0^h$$

$$\delta = \frac{Ph}{Et(d_2 - d_1)} \ln \left(\frac{d_2}{d_1}\right)$$