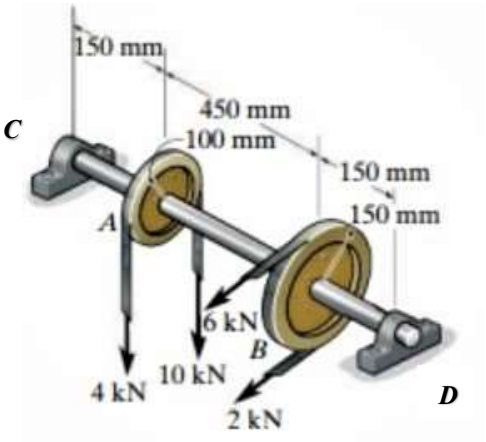
MEC85 / CEC30 Midterm 2 – Spring 2019
Professor Grace X. Gu

Name: SID:

Total Points: 40 (3 Problems)
Show all your work and **write neatly**. Clearly state the direction of the forces you evaluate. Partial credit will be given. Good luck!

Problem 1 (8 + 2 = 10 points)

a) The central shaft which is used to transmit power in a mechanical device has a diameter of $\frac{20}{3}$ cm and is supported at its two ends C and D by bearings that allow the shaft to rotate freely. The shaft is made up of three materials fused together – section C-A has a shear modulus G = 64 GPa, section A-B has a shear modulus G = 81 GPa, and section B-D has a shear modulus G = 100 GPa. Determine the angle of twist of wheel B with respect to wheel A. You can leave your final answer in terms of π.



b) The same arrangement is again set up for a shaft made of a single material with an unknown but constant G. Give a brief explanation of how you would determine G experimentally.

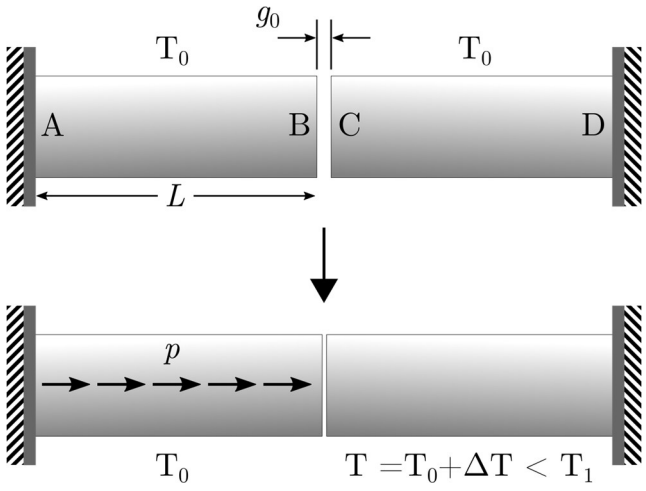
Problem 2 (20 points)

Consider two identical weightless coaxial rods AB and CD, with length L, area A, Young's modulus E and linear coefficient of thermal expansion α. Each is built into a rigid support at one end (A and D), and the rods are aligned such that the free ends (B and C) are initially separated by a small gap $g_0 \ll L$.

Rod AB is loaded by a uniform distributed axial force p (with physical dimension of force per unit length). Rod AB is held at ambient temperature T_0 throughout this problem. We take p to be small enough that the deflection of end B is less than the initial gap ($\delta_B < g_0$).

Rod CD is subjected to a temperature increase of ΔT from the ambient temperature. No external forces are applied to CD.

Let T_1 be the temperature at which the ends of the rods first touch due to the deflections of the ends B and C, and let $\Delta T_1 = T_1 - T_0$.



a) At a temperature just below T_1, (just before the ends touch), determine the axial stresses in the rods at points A and D.

b) Derive an equation for ΔT_1 in terms of the parameters given: L, A, g_0, E, α, p and T_0. (Note: your equation may or may not involve all of these parameters.)

Now let the temperature increase to $T_2 > T_1$ so that the ends of the rods push against one another. As before, let $\Delta T_2 = T_2 - T_0$, but also let $\Delta T_2^* = T_2 - T_1$.

c) At this increased temperature T_2, determine the axial force that the rods exert on one another. Express your answer in terms of the parameters given above: $L, A, g_0, E, \alpha, p, T_0, T_1, T_2, \Delta T_1, \Delta T_2$ and ΔT_2^*. Your answer need not involve all of these parameters.

Problem 3 (10 points)

A tapered plate with an elastic modulus E is subjected to an axial load P as shown below. Calculate the displacement of one end of the plate relative to the other in terms of the given quantities P, d_1, d_2, h, t and E.