EE 120: Signals and Systems
Department of Electrical Engineering and Computer Sciences — MIDTERM 2
UC BERKELEY — 21 March 2019

FIRST Name ______________ LAST Name ______________

Discussion Section Time: __________ SID (All Digits): __________

- **(10 Points)** Print your *official* name (not your e-mail address) and *all* digits of your student ID number legibly, and indicate your lab time, on *every* page.
- This exam should take up to 100 minutes to complete. However, you may use up to a maximum of 110 minutes *in one sitting*, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except two double-sided $8.5" \times 11"$ sheets of handwritten, original notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- We will provide you with scratch paper. Do not use your own.
- **The exam printout consists of pages numbered 1 through 10.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the ten numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't evaluate it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

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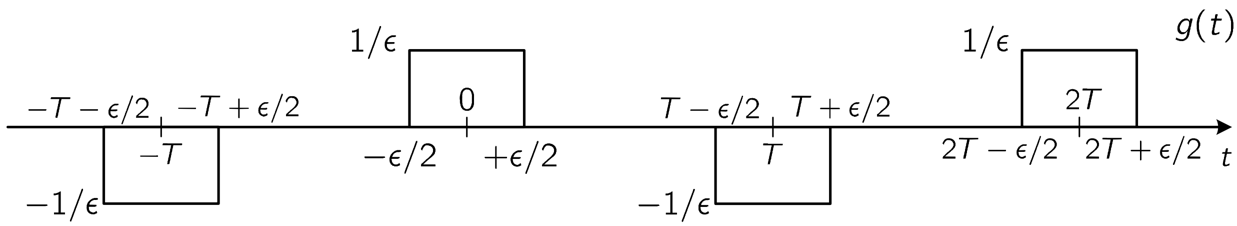
MT2.1 (70 Points) Consider the periodic continuous-time signal g shown in the figure below. The alternating square pulse pattern repeats outside the portion of the time axis depicted by the figure. And the parameter T is sufficiently large; in particular, assume $2\epsilon < T$.



(a) (10 Points) Determine the fundamental period p and the fundamental frequency ω_0 of the signal g.

(b) (20 Points) Determine a reasonably simple expression for the exponential Fourier series coefficients G_k (for all k in $\mathbb{Z}$) of the signal g.

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(c) (20 Points) Suppose $\epsilon \to 0$ in the figure above. Provide a well-labeled plot of the signal g in this limiting case and determine the corresponding exponential Fourier series coefficients G_k for g. You should be able to tackle this part even if you did not get through, or are not confident in your answer to, part (b).

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(d) (20 Points) Suppose g (with $\epsilon > 0$) is the impulse response of an LTI system whose input signal x is defined by the following:

$$\forall t, \quad x(t) = \begin{cases} g(t) & |t| < T/2 \\ 0 & \text{elsewhere.} \end{cases}$$

Provide a well-labeled plot of the output signal y corresponding to this input signal x. Remember that T is sufficiently large.

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MT2.2 (30 Points) DTFS . . .

(a) (15 Points) Consider a positive integer p, and let $\omega_0 = 2\pi/p$. Prove that if $k \bmod p \neq 0$, then

$$\sum_{n=\langle p\rangle} \cos(k\omega_0 n) = \sum_{n=\langle p\rangle} \sin(k\omega_0 n) = 0.$$

Hint: Let p and ω_0 be the fundamental period and frequency, respectively, of a discrete-time signal x. Take advantage of what you know about the orthogonality of the basis functions Ψ_k in the complex exponential DTFS expansion

$$x(n) = \sum_{k=\langle p\rangle} X_k \underbrace{e^{ik\omega_0 n}}_{\Psi_k(n)}.$$

(b) (15 Points) Determine the smallest positive integers P_1 and P_2 such that each of the following statements is true; explain your reasoning even if you believe no finite integer value can be found.

$$\sum_{n=\langle P_1\rangle} \cos\left(\frac{5\pi n}{3}\right) = 0 \qquad\qquad \sum_{n=\langle P_2\rangle} e^{in} = 0.$$

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MT2.3 (30 Points) The input x and corresponding output y of a continuous-time system G are

$$\forall t \in \mathbb{R}, \quad x(t) = \sum_{\ell=-\infty}^{+\infty} \delta(t-\ell) \quad \text{and} \quad y(t) = \sum_{m=-\infty}^{+\infty} (-1)^m \delta(t-m).$$

(a) (10 Points) Select the strongest assertion, which is true, from the choices below (and fill in the blank, as appropriate). Explain your choice succinctly, but clearly and convincingly.

  (i) G must be an LTI system, and its frequency response is ______.

  (ii) G can be an LTI system, and if it is, its frequency response is ______.

  (iii) G cannot be an LTI system.

(b) (20 Points) Determine the complex exponential Fourier series expansion of the output y.

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MT2.4 (40 Points) Consider a discrete-time causal LTI filter H whose input x and output y satisfy the linear, constant-coefficient difference equation

$$y(n) = \alpha\, y(n-4) + x(n) - x(n-4), \tag{1}$$

where the parameter $\alpha = (0.95)^4$.

(a) (20 Points) Determine a reasonably simple expression for $H(\omega)$, the frequency response of the system, and provide a well-labeled plot of the magnitude response $|H(\omega)|$. You must explain any reasonable approximation you make to plot the magnitude response.

(b) (20 Points) The fundamental period of an input to the system is $p = 8$. If the DTFS expansion of the input signal is given by $x(n) = \sum_{k=0}^{p-1} X_k\, e^{ik\omega_0 n}$, determine the fundamental frequency ω_0 and a reasonable expression for the DTFS expansion of the output y.

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MT2.5 (20 Points) Consider a periodic discrete-time signal g described as follows:

$$\forall n, \quad g(n) = \begin{cases} +1 & n = 0, \pm 6, \pm 12, \ldots \\ -1 & n = \pm 3, \pm 9, \ldots \\ 0 & \text{elsewhere.} \end{cases}$$

Determine the discrete-time Fourier series coefficients G_k for this signal.

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FORMULAS & TABLES

Discrete-Time Fourier Series (DTFS) Complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period p:

$$x(n) = \sum_{k=\langle p \rangle} X_k \, e^{ik\omega_0 n} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n) \, e^{-ik\omega_0 n} \,,$$

where $p = \dfrac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable discrete interval of length p (i.e., an interval containing p contiguous integers). For example, $\sum_{k=\langle p \rangle}$ may denote $\sum_{k=0}^{p-1}$ or $\sum_{k=1}^{p}$.

Continuous-Time Fourier Series (CTFS) Complex exponential Fourier series synthesis and analysis equations for a periodic continuous-time signal having period p:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{ik\omega_0 t} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \int_{\langle p \rangle} x(t) \, e^{-ik\omega_0 t} dt \,,$$

where $p = \dfrac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable continuous interval of length p. For example, $\int_{\langle p \rangle}$ can denote $\int_0^p$.