

Spring 2005 MSE111 Midterm

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3/15/05, 9:40 am

**University of California at Berkeley
Department of Materials Science and Engineering**

80 minutes, 68 points total, 10 pages

Name: _____

SID: _____

Problem	1	2	3	4	5	TOTAL
Points Possible	7	23	10	12	16	68
Points						

SHOW ALL OF YOUR WORK!!!

Answers given without supporting calculations will be marked wrong, even if they are numerically correct.

1. (7 pts.) True/False

CIRCLE *T* or *F* indicating whether the statement is true or false

- 1) Germanium is metallically bonded. **T F**
- 2) Metallic bonds are localized. **T F**
- 3) Two copper pieces of the same purity but of different dimensions will have different resistivities. **T F**
- 4) Phonons obey the Pauli exclusion principle. **T F**
- 5) Metals have forbidden band gaps. **T F**
- 6) In a Hall Effect measurement of a metal, the carrier mobility must be known to determine the carrier concentration. **T F**
- 7) Phonon energies of Si are much smaller than the band gap energy. **T F**

2. (23 pts.) Short Answers

Answer each question within 50 words.

a) (4 pts.) The mass action law of semiconductors is $np = n_i^2$. At what temperature does it apply? Is it valid for extrinsic semiconductors?

b) (2 pts.) Explain the difference between extrinsic and intrinsic semiconductors.

c) (4 pts.) At a temperature of 100K and an Fermi energy of 5eV, what is the probability of finding an electron at 4.5 eV?

d) (4 pts.) For the same system in part c), what is the probability of finding a hole at 4.5eV?

e) (2 pts.) At what temperature is there a 50% probability that a state with an energy level at the Fermi energy will be occupied by an electron?

f) (7 pts.) In intrinsic semiconductors the concentration of free carriers is a strong function of temperature. The intrinsic carrier concentration in Si ($E_g=1.1\text{eV}$) at 300K (room temperature) is $2 \times 10^{10} \text{ cm}^{-3}$. How much should the Si be cooled to reduce this concentration by three orders of magnitude?

3. (10 pts.) Davisson and Germer (Electron Diffraction)

An electron beam impinges **perpendicularly** on a Si (100) surface. **Top layer surface diffraction** is observed.

a) **(5 pts.)** Make a sketch and derive the condition for constructive interference from top layer diffraction in terms of the wavelength of incoming electrons λ , distance between neighboring Si atoms d and the angle between the incoming beam and the diffracted beam θ . (**Hint:** Only the top atomic layer of Si atoms contribute to the diffraction!)

b) **(5 pts.)** Use your result in part a) to find the energy of an electron that is diffracted at an angle 25° off this (100) surface. Assume the distance between Si atoms to be $d_{\text{Si}} = 3.840 \text{ \AA}$. Assume only 1st order diffraction happens. (**Hint:** Use Bragg's Law if you couldn't get the final result in part a).)

4. (12 pts.) Bonding

The bonding potential can be approximated for many cases by the function

$$E(r) = \frac{A}{r^n} - \frac{B}{r^m}$$

a) (4 pts.) Sketch the energy E vs separation r for an ionic bond between two atoms of opposite charge, making sure to label the bond length r_0 and the bond energy E_c :



b) (2 pts.) Explain the origins of the forces that act on the two ions to give the curve its shape in two sentences.

c) (4 pts.) What is the condition for approximating an ionic bond to a spring that obeys Hooke's Law ($F=kx$)? Explain your answer in terms of the E vs. r sketch and state which feature of this sketch characterizes the “stiffness” of the bond.

d) (2 pts.) What is r_0 in terms of A , B , n and m ?

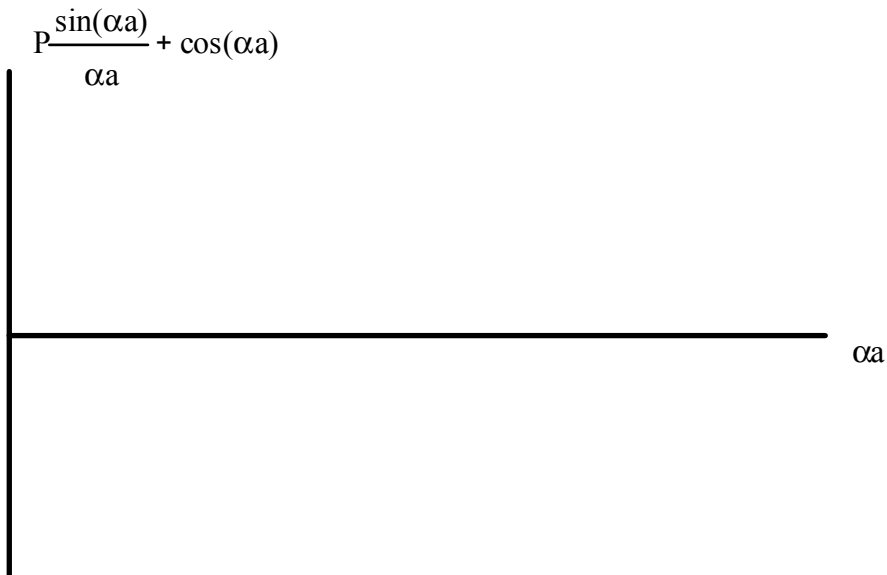
5. (15 pts.) Kronig-Penney Model

A condition for the solution of the Schrödinger equation for the Kronig-Penney model of a one-dimensional solid is given by

$$P \frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a) = \cos(ka) \quad \text{Eq. 1}$$

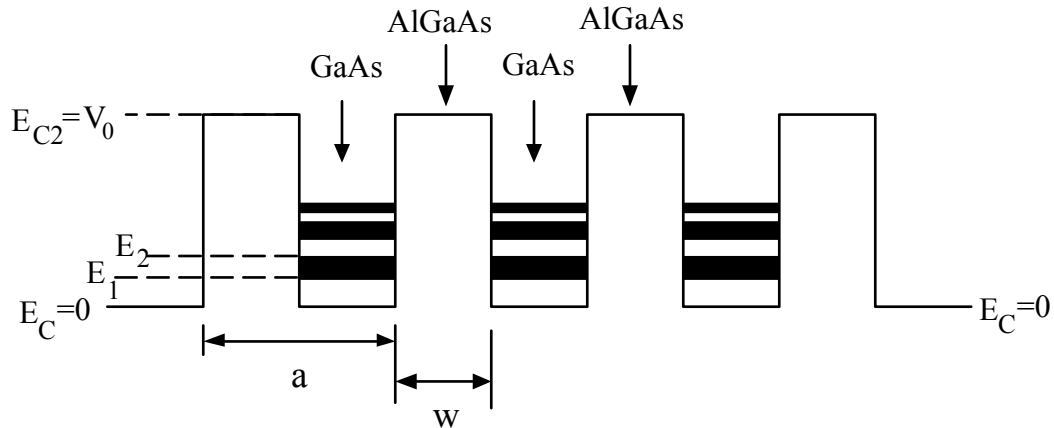
where $P = \frac{m^* a}{\hbar^2} V_0 w$ and $\alpha = \sqrt{\frac{2m^* E}{\hbar^2}}$

a) (5 pts.) Sketch the left-hand side of Eq. 1 on the axes given below. Indicate the 1st and 2nd allowed bands and the 1st forbidden band.



5. Continued... Kronig-Penney Model

The Kronig-Penney model can be used to describe to first order the nature of the conduction band in superlattice structures.



A conduction electron is exposed to a periodic potential formed by the different bandgaps and their offsets between GaAs and $Al_{1-x}Ga_xAs$ (denoted by AlGaAs in the figure above). The black regions within the GaAs conduction band depict the bands of allowed energy, or “mini-bands,” as determined by the Kronig-Penney model. The band demarcated by E_1 and E_2 refers to the first allowed energy band as calculated from the equation on the previous page. Assume that the conduction band edge in the GaAs region is taken as the reference point, (i.e., $E_C = 0$) and that $w = 40 \text{ \AA}$, $a = 80 \text{ \AA}$, $m^* = 0.07m_e$, and $E_1 = 0.065 \text{ eV}$ for which $\alpha a = 2.77$.

5. Continued... Kronig-Penney Model

b) (4 pts.) Find E_2 (for which $\alpha a = \pi$). Give your answer in eV.

c) (5 pts.) What is the magnitude of the barrier, (i.e., $E_{C2} = V_0$)? (**Hint:** solve for P by considering Eq. 1 at $\alpha a = 2.77$, where the first allowed mini-band begins.)

d) (2 pts.) If an electron spends 10^{-12} seconds within the first mini-band, what is the uncertainty with which its energy can be measured?

Constants

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$e = -1.6 \times 10^{-19} \text{ C}$$

$$h = 6.625 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$1 \text{ \AA} = 1 \times 10^{-10} \text{ m}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$k_b = 8.62 \times 10^{-5} \text{ eV/K}$$

$$m_{\text{proton}} = 1.67 \times 10^{-27} \text{ kg}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$N_A = 6.02 \times 10^{23} \text{ atoms/mole}$$

Equations:

$$\Delta p \Delta x \geq h \quad \Delta E \Delta t \geq h$$

$$E = \frac{p^2}{2m} = \frac{1}{2} m v^2$$

$$R = \rho \frac{L}{A}$$

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{2\pi}{k}$$

$$E = \frac{h c}{\lambda} = h \nu$$

$$E_n = \frac{n^2 h^2}{8mL^2}$$

$$\sigma = e(n\mu_e + p\mu_h)$$

$$\mu = \frac{e\tau}{m}$$

$$\sigma = \frac{1}{\rho}$$

$$J = NV_d e = N\mu E e$$

$$E_H = R_H J B$$

$$R_H = \frac{1}{eN_e}$$

time-independent Schrödinger equation:

$$\left(\frac{-\hbar^2}{2m} \right) \frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$$

common solutions:

$$\psi(x) = [A \exp(ikx) + B \exp(-ikx)] \quad \psi(x) = C \exp(kx) + D \exp(-kx)$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$E_n = \frac{-13.6}{n^2}$$

$$Z(E) dE = \frac{4\pi L^3 (2m)^{3/2}}{h^3} E^{1/2} dE$$

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2k_b T}\right)$$

$$F(E) = \frac{1}{1 + \exp\left[\frac{(E - E_F)}{k_b T}\right]}$$

$$p = N_v \exp\left(-\frac{E_F}{k_b T}\right)$$

$$n = N_c \exp\left(-\frac{(E_g - E_F)}{k_b T}\right)$$

$$\omega = \left(\frac{4C}{M}\right)^{1/2} \left| \sin\left(\frac{ka}{2}\right) \right|$$

$$NL^x = \int_0^\infty Z(E) F(E) dE \quad x=1,2,3$$

$$N_v = 2 \left(\frac{2\pi m_h^* k_b T}{h^2} \right)^{3/2}$$

$$E_F = \frac{E_g}{2} + \frac{3}{4} kT \ln \frac{m_h^*}{m_e^*}$$

$$m^* = \left(\frac{h}{2\pi} \right)^2 \left(\frac{\partial^2 E}{\partial k^2} \right)^{-1}$$