Solutions to Midterm 1

Problem 1. A normal acceleration is required for curvilinear motion. In top position B,

$$\sum F_n = ma_n$$

$$\Rightarrow \quad mg - N_B = m\frac{v^2}{\rho}$$

$$\Rightarrow \quad N_B = 2g - 2\frac{3.5^2}{2.4} = 9.41 \text{ N}$$

In position A,

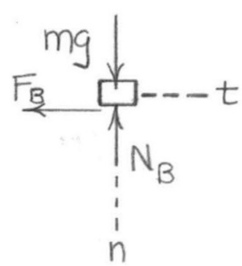
$$\sum F_n = ma_n$$

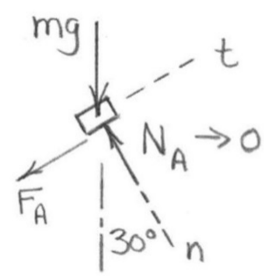
$$\Rightarrow \quad mg\cos 30° - N_A = m\frac{v^2}{\rho}$$

$$\Rightarrow \quad N_A = mg\cos 30° - m\frac{v^2}{\rho}$$

As v increases, N_A decreases. When the block is about to lose contact, $N_A = 0$ and

$$v_{\max} = (\rho g \cos 30°)^{1/2} = 4.52 \text{ m/s}$$





Problem 2. The extension of the spring in position A is

$$e = \sqrt{2}R - R$$

When the slider moves from initial rest position at A to a new position at B,

$$U_{1-2} = \Delta T = T_2$$

$$\Rightarrow \quad mgR + \frac{1}{2}ke^2 = \frac{1}{2}mv_B^2$$

$$\Rightarrow \quad v_B = \sqrt{2gR + \frac{kR^2}{m}(3 - 2\sqrt{2})}$$

Similarly, if the slider moves from initial rest position at A to a new position at C,

$$U_{1-2} = \Delta T = T_2$$

$$\Rightarrow \quad mg(2R) + \frac{1}{2}ke^2 = \frac{1}{2}mv_C^2$$

$$\Rightarrow \quad v_C = \sqrt{4gR + \frac{kR^2}{m}(3 - 2\sqrt{2})}$$

At position C, the normal force N exerted by the guide on the slider is directed vertically upwards.

$$\sum F_n = ma_n$$

$$\Rightarrow \quad N - mg = m\frac{v_C^2}{R}$$

$$\Rightarrow \quad N = m\left[5g + \frac{kR}{m}(3 - 2\sqrt{2})\right]$$

Problem 3. When plug A moves from initial rest position at B to a new position just before striking block C,

$$U_{1-2} = \Delta T = T_2$$

$$\Rightarrow \quad m_A g r = \frac{1}{2} m_A v^2$$

$$\Rightarrow \quad v = \sqrt{2gr}$$

Let v' be the horizontal velocity of plug and the block after impact. Then

$$\Delta G = 0$$

$$\Rightarrow \quad m_A v = (m_A + m_C) v'$$

$$\Rightarrow \quad v' = \frac{m_A}{m_A + m_C} \sqrt{2gr}$$

After impact, the plug and block are subjected to a limiting friction

$$F = \mu_k (m_A + m_C) g$$

This frictional force produces a constant deceleration

$$a = \frac{F}{m_A + m_C} = \mu_k g$$

The distance s which the block and plug slide before coming to rest is given by

$$v'^2 = 2as$$

$$\Rightarrow \quad s = \left(\frac{m_A}{m_A + m_C}\right)^2 2gr \frac{1}{2\mu_k g} = \frac{r}{\mu_k} \left(\frac{m_A}{m_A + m_C}\right)^2$$