Physics 7A - Midterm 1 Solution
Fall 2018 (Hallatschek)
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1. (a) Horizontally, the car moves at a constant speed $v_0 \cos\theta$, while vertically it has an initial speed $v_0 \sin\theta$ and experiences a negative acceleration g. So

$$\boxed{x = v_0 \cos\theta t,\ y = v_0 \sin\theta t - \frac{1}{2} g t^2.}$$

   (b) To make the car land at point $B\,(L, 0)$, we set $x = L$ and $y = 0$ in the two equations above, and eliminate t. It follows that $v_0 = \boxed{\sqrt{gL/\sin 2\theta}.}$

2. (a) There are four pieces of rope pulling up the system (man+plate), and each rope has the same tension F. So $4F - (M+m)\,g = (M+m)\,a$, which gives $a = \boxed{4F/(M+m) - g.}$

   (b) If the man is lifted above the plate, only two pieces of ropes are pulling the man and the other two are pulling the plate. So we have $2F - Mg = Ma_M$ and $2F - mg = ma_m$. Therefore $a_M = \boxed{2F/M - g,}$ and $a_m = \boxed{2F/m - g.}$

   Note that in (a) there is a normal force between m and M, while in (b) the normal force is absent. This makes the two scenarios different.

3. Let (x_M, y_M) be the position of the top corner of the wedge M, and (x_m, y_m) be the position of the block m. Since the block must move along the surface of the wedge, the displacement of m relative to M must "fit" the surface, as shown in the left figure below. Namely, the **constraint** of the motion is

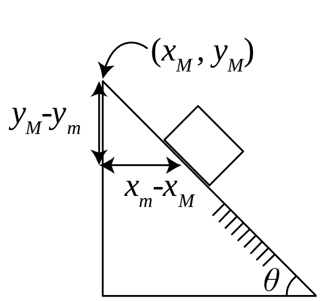
$$\tan\theta = \frac{y_M - y_m}{x_m - x_M},\ \text{or}\ y_M - y_m = (x_m - x_M)\tan\theta.$$

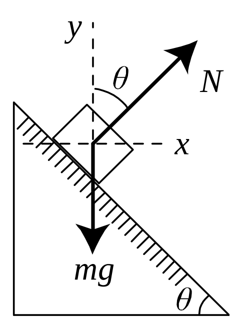
Differentiating the equation above with respect to time twice, and using the fact that $\ddot{x}_M = a$, $\ddot{y}_M = 0$, we get $-\ddot{y}_m = (\ddot{x}_m - a)\tan\theta$. From the free body diagram below, we also have:

$$N\sin\theta = m\ddot{x}_m,\ \text{and}\ N\cos\theta - mg = m\ddot{y}_m.$$

Eliminating N from these two equations, and using $\theta = \pi/4$, we solve for $\ddot{x}_m$ and $\ddot{y}_m$:

$$\ddot{x}_m = \frac{a+g}{2},\ \ddot{y}_m = \frac{a-g}{2}$$





4. See the free body diagram below. Since the car is climbing up a circle of radius R using the static friction force f_s, and at the same time rotating about the center with speed v, it follows that

$$\begin{aligned} N - mg\cos\theta &= \frac{mv^2}{R}, \\ mg\sin\theta &= f_s. \end{aligned}$$

Not falling means $N \geq 0$, and not slipping implies $f_s \leq \mu_s N$. Imposing these two conditions to the two equations above, we get

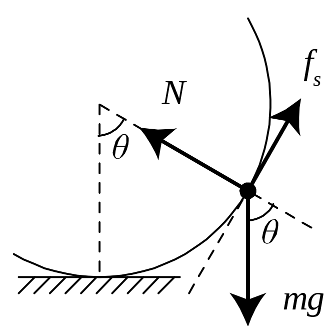
$$\begin{cases} \cos\theta + \dfrac{v^2}{gR} \geq 0 & \text{(not falling)}, \\ \mu_s \geq \dfrac{\sin\theta}{\cos\theta + v^2/gR} & \text{(not slipping)}, \end{cases} \quad \text{for all } \theta.$$

Since $\cos\theta$ can be as negative as -1 (when $\theta = \pi$, i.e., at the top of the circle), to ensure the first inequality to hold, we must require $v \geq \sqrt{gR}$. (To ensure the second inequality to hold, we need both $v > \sqrt{gR}$, and $\mu_s \geq 1/\sqrt{(v^2/gR)^2 - 1}$. But we don't need these results for the exam.)

(a) From the analysis above, the car has to move at a speed $\boxed{v > v_0 = \sqrt{gR}}$ to not fall off from the track.

(b) If $v < v_0 = \sqrt{gR}$, the car starts to slip when

$$\boxed{\mu_s = \frac{\sin\theta}{\cos\theta + v^2/gR}}.$$

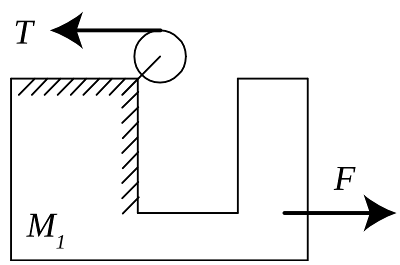


5. (a) The mass M_3 is pulling the rope connecting M_2 and M_3, so both M_2 and M_3 have the same acceleration a. The equation of motion for M_2 and M_3 are $T = M_2 a$ and $M_3 g - T = M_3 a$, respectively. Solving these two equations for a and T, we obtain

$$a = \boxed{\frac{M_3 g}{M_2 + M_3},} \text{ and } T = \boxed{\frac{M_2 M_3}{M_2 + M_3} g.}$$

In the free body diagram of M_1 below, this tension T is *pushing* M_1 to the left, so the force F has to oppose it to make M_1 still:

$$F = \boxed{\frac{M_2 M_3 g}{M_2 + M_3}.}$$



(b) The tension T has to balance M_3 in the vertical direction, so $T = M_3 g$. But the same T also needs to pull M_2 such that M_2 has the same acceleration as the entire system, so $T = M_2 a$. Hence $a = M_3 g / M_2$. The force F that moves entire system $M_1 + M_2 + M_3$ with acceleration a should have magnitude

$$F = \boxed{(M_1 + M_2 + M_3) \frac{M_3 g}{M_2}.}$$

(c) Assume M_1 has acceleration $-a\hat{\mathbf{e}}_x$, and M_2 has acceleration $a'\hat{\mathbf{e}}_x$, both relative to ground. Then relative to M_1, the mass M_2 has acceleration $(a' + a)\,\hat{\mathbf{e}}_x$. Since the rope has fixed length, $a' + a$ is also the downward acceleration of M_3. Therefore, we can write

$$\begin{aligned} T &= M_2 a', \\ M_3 g - T &= M_3 (a' + a). \end{aligned}$$

Eliminate a', we get $T = M_2 M_3 (g - a) / (M_2 + M_3)$. This tension is pushing M_1 and M_3 (via the wall next to M_3) to the left with acceleration $-a\hat{\mathbf{e}}_x$, so

$$\frac{M_2 M_3}{M_2 + M_3} (g - a) = (M_1 + M_3)\, a.$$

Solving this equation for a, it follows that

$$a = \boxed{\frac{\mu g}{1 + \mu},} \text{ where } \mu \equiv \frac{M_2 M_3}{(M_1 + M_3)(M_2 + M_3)}.$$