

EE 120: SIGNALS AND SYSTEMS, FALL 2019

Midterm 1, October 3, Thursday, 12:10-2:00 pm

Name: _____ *Solutions*

SID #: _____

Important Instructions:

Closed book. As announced before, you can use one letter-size, two-sided cheatsheet.

Show all your work. An answer without explanation is not acceptable and does not guarantee any credit.

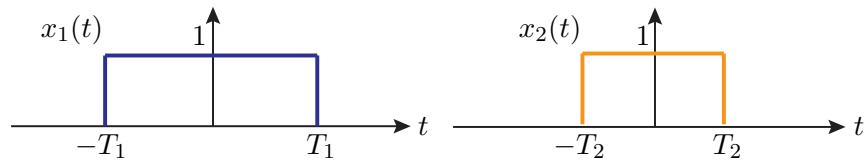
You can use the back of the pages if you need more space.

Do not remove pages, as this disrupts the scanning. Instead, cross the parts that you don't want us to grade.

Problem	Points
1	20
2	20
3	20
4	20
5	20
Total	100

1. (Convolution)

a) (10 points) Find the convolution $x_1 * x_2$ for the signals x_1 and x_2 below, where $T_1 > T_2$. Plot and clearly label $(x_1 * x_2)(t)$.



b) (5 points) Express the integral $\int_{-\infty}^{t-1} x(\tau) d\tau$ as a convolution $(x * g)(t)$ with a signal g that you will determine.

c) (5 points) Find a continuous-time signal x , not identically equal to zero, such that $x * x = x$.

2. (LTI Systems) Consider a LTI system defined by the difference equation:

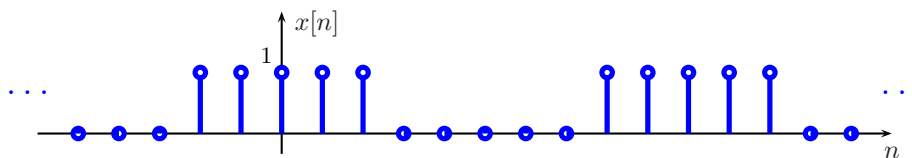
$$y[n] - 1.1y[n - 1] + 0.3y[n - 2] = 2x[n] - x[n - 1]. \quad (1)$$

- a) (6 points) Find the frequency response $H(e^{j\omega})$ for (1).
- b) (8 points) Find the impulse response $h[n]$ for (1).
- c) (6 points) Determine if (1) is a stable system.

Additional space for Problem 2.

3. (Fourier Series) Given the period-10 sequence $x[n]$ depicted below, determine the following quantities where a_k denotes the k th Fourier series coefficient:

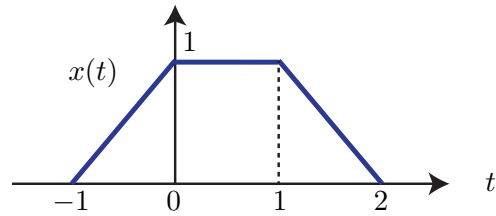
- a) (4 points) a_0 ,
- b) (4 points) a_5 ,
- c) (4 points) Imaginary part of a_7 .
- d) (4 points) $\sum_{k=0}^9 a_k$,
- e) (4 points) $\sum_{k=0}^9 (-1)^k a_k$.



Additional space for Problem 3.

4. (Properties of the Fourier Transform) To answer the following questions you can use relevant properties of the Fourier Transform along with Fourier Transform pairs already derived in class.

a) (5 points) Find the Fourier Transform of the signal shown below.



b) (5 points) Find the Fourier Transform of the signal:

$$x(t) = \begin{cases} \cos(10\pi t) & |t| \leq 0.5 \\ 0 & |t| > 0.5. \end{cases}$$

c) (5 points) Evaluate the integral:

$$\int_{-\infty}^{\infty} \text{sinc}^2(t) dt.$$

d) (5 points) Prove the following property of convolution:

$$(f * g)' = f' * g = f * g'$$

where $'$ represents the derivative: $f'(t) := df(t)/dt$.

5. Define the cosine transform by:

$$X^{\cos}(\omega) := \int_{-\infty}^{\infty} x(t) \cos(\omega t) dt.$$

a) (7 points) Show that, when x is real valued, the cosine transform is the real part of the Fourier Transform: $X^{\cos}(\omega) = \operatorname{Re}\{X(\omega)\}$.

b) (6 points) Show that the cosine transform is linear. In other words, for any two signals x_1, x_2 , and constants α, β , the cosine transform of $\alpha x_1 + \beta x_2$ is $\alpha X_1^{\cos} + \beta X_2^{\cos}$, where X_i^{\cos} is the cosine transform of x_i , $i = 1, 2$.

c) (7 points) Show that $X^{\cos}(\omega) = 0$ for all ω when x is real and odd-symmetric: $x(-t) = -x(t)$ for all t .

