EE 120: SIGNALS AND SYSTEMS, FALL 2019

Midterm 1, October 3, Thursday, 12:10-2:00 pm

Name: ________________

SID #: ________________

Important Instructions:

**Closed book.** As announced before, you can use one letter-size, two-sided cheatsheet.

**Show all your work.** An answer without explanation is not acceptable and does not guarantee any credit.

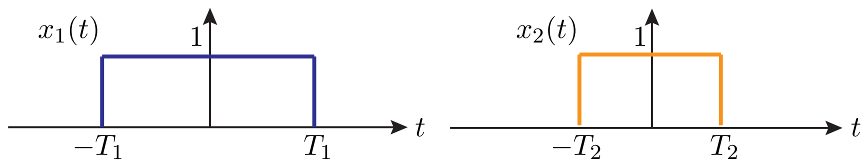
**You can use the back of the pages** if you need more space.

**Do not remove pages**, as this disrupts the scanning. Instead, cross the parts that you don't want us to grade.

| Problem | Points |
|---|---|
| 1 | 20 |
| 2 | 20 |
| 3 | 20 |
| 4 | 20 |
| 5 | 20 |
| Total | 100 |

1. (Convolution)

a) (10 points) Find the convolution $x_1 * x_2$ for the signals $x_1$ and $x_2$ below, where $T_1 > T_2$. Plot and clearly label $(x_1 * x_2)(t)$.



b) (5 points) Express the integral $\int_{-\infty}^{t-1} x(\tau)d\tau$ as a convolution $(x * g)(t)$ with a signal $g$ that you will determine.

c) (5 points) Find a continuous-time signal $x$, not identically equal to zero, such that $x * x = x$.

2. (LTI Systems) Consider a LTI system defined by the difference equation:

$$y[n] - 1.1y[n-1] + 0.3y[n-2] = 2x[n] - x[n-1]. \tag{1}$$

a) (6 points) Find the frequency response $H(e^{j\omega})$ for (1).
b) (8 points) Find the impulse response $h[n]$ for (1).
c) (6 points) Determine if (1) is a stable system.

Additional space for Problem 2.

3. (Fourier Series) Given the period-10 sequence $x[n]$ depicted below, determine the following quantities where $a_k$ denotes the $k$th Fourier series coefficient:

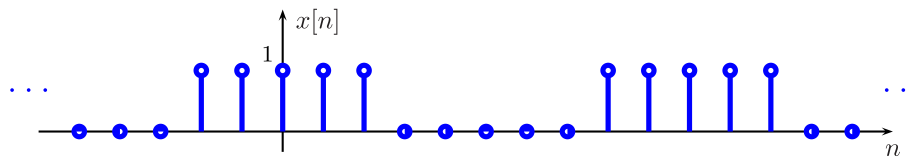
a) (4 points) $a_0$,

b) (4 points) $a_5$,

c) (4 points) Imaginary part of $a_7$.

d) (4 points) $\sum_{k=0}^{9} a_k$,

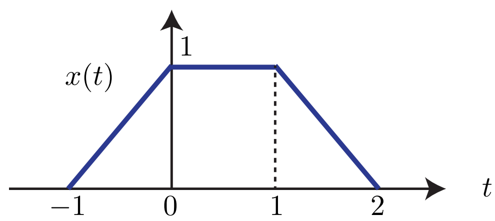
e) (4 points) $\sum_{k=0}^{9} (-1)^k a_k$.



Additional space for Problem 3.

4. (Properties of the Fourier Transform) To answer the following questions you can use relevant properties of the Fourier Transform along with Fourier Transform pairs already derived in class.

a) (5 points) Find the Fourier Transform of the signal shown below.



b) (5 points) Find the Fourier Transform of the signal:

$$x(t) = \begin{cases} \cos(10\pi t) & |t| \leq 0.5 \\ 0 & |t| > 0.5. \end{cases}$$

c) (5 points) Evaluate the integral:

$$\int_{-\infty}^{\infty} \mathrm{sinc}^2(t)dt.$$

d) (5 points) Prove the following property of convolution:

$$(f * g)' = f' * g = f * g'$$

where $'$ represents the derivative: $f'(t) := df(t)/dt$.

5. Define the cosine transform by:

$$X^{\cos}(\omega) := \int_{-\infty}^{\infty} x(t)\cos(\omega t)dt.$$

a) (7 points) Show that, when $x$ is real valued, the cosine transform is the real part of the Fourier Transform: $X^{\cos}(\omega) = \mathrm{Re}\{X(\omega)\}$.

b) (6 points) Show that the cosine transform is linear. In other words, for any two signals $x_1$, $x_2$, and constants $\alpha$, $\beta$, the cosine transform of $\alpha x_1 + \beta x_2$ is $\alpha X_1^{\cos} + \beta X_2^{\cos}$, where $X_i^{\cos}$ is the cosine transform of $x_i$, $i = 1, 2$.

c) (7 points) Show that $X^{\cos}(\omega) = 0$ for all $\omega$ when $x$ is real and odd-symmetric: $x(-t) = -x(t)$ for all $t$.