mean: 128
std: 53
median: 140

UNIVERSITY OF CALIFORNIA, BERKELEY
MECHANICAL ENGINEERING
ME106 Fluid Mechanics **SOLUTIONS**
2nd Test, S19 *Prof S. Morris*

1. **(100)**For an ideal gas having constant specific heat c_p, $\frac{1}{2}V^2 + c_pT$ is constant along a streamline in steady isentropic flow. Using that Bernoulli relation, and the isentropic relations for an ideal gas, and assuming subsonic flow, derive the expression giving the flow speed V in terms of the pressure and the temperature at the stagnation point O and the pressure at point A on the Pitot–static tube illustrated. Why would your expression not hold if the flow were supersonic?

Solution

10 10

By combining the ideal gas law $p = \rho RT$ and the isentropic relation $p \propto \rho^\gamma$, 40

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{1-1/\gamma}, \quad 20 \tag{1.1}$$

where p_0 and T_0 can be chosen to be evaluated at O.

20

By applying the Bernoulli relation along the streamline from O to A, $\frac{1}{2}V^2 + c_pT = c_pT_0$, and by using (1.1) to eliminate T/T_0,

$$V_A^2 = 2c_pT_0\left\{1 - \left(\frac{p_A}{p_0}\right)^{1-1/\gamma}\right\} \quad 20 \tag{1.2}$$

Because streamlines are parallel at A, the normal component of the Euler equation requires p to be constant across streamlines. At infinity, far above A, speed V and pressure p are identical with those in the undisturbed flow at infinity to the left of O. By measuring T and p at O and p at A, V_A can be found from (1.2).

For supersonic flow, the Pitot–static tube is separated from the flow at infinity by a shock wave. The formula (1.2) must then be modified to account for the entropy change across the shock wave. 20

1 mean: 67
std: 29
median: 80

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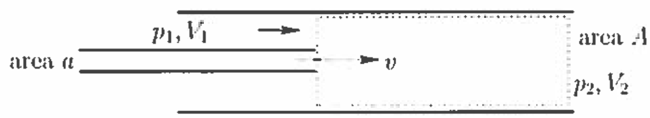
2. (100) High speed fluid of density ρ is injected with speed v through the small tube as a free jet into a larger tube containing the same fluid moving with speed $V_1 < v$. Downstream of the mixing region, the velocity and the pressure can both be assumed to be uniform across the tube. The flow is incompressible and the Reynolds number is large. (a) By balancing mass and momentum on the control volume illustrated, show that

$$p_2 - p_1 = \rho\alpha(1-\alpha)(v - V_1)^2,$$

where the area ratio $\alpha = a/A$.

60 (b) Show that between points 1 and 2, the power loss

$$\text{P.L.} = \frac{1}{2}(p_2 - p_1)A\big([1-\alpha]v + \alpha V_1\big).$$



Given: (a) Provided the control surface can be chosen so that, at any point on it, viscous stresses do negligible work,

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{CV}\rho\Big\{\frac{1}{2}V^2 + gz\Big\}\,\mathrm{d}V + \oint_{CS}\Big\{\frac{1}{2}V^2 + gz + \frac{p}{\rho}\Big\}(\rho\mathbf{V}\cdot\mathbf{n})\,\mathrm{d}S = \text{S.P.} - \text{P.L.}$$

(b) An identity: $\alpha v^3 + (1-\alpha)V_1^3 - \big\{\alpha v + (1-\alpha)V_1\big\}^3 = \alpha(1-\alpha)(v - V_1)^2\big\{(1+\alpha)v + (2-\alpha)V_1\big\}$.

<u>Solution</u>

60 (a) By balancing mass and momentum on the control volume shown, $AV_2 = (A-a)V_1 + av$ and $(p_1 - p_2)A = \rho\big\{(AV_2^2 - (av^2 + (A-a)V_1^2)\big\}$, which can be written

$$V_2 = (1-\alpha)V_1 + \alpha v, \qquad \text{mass: 20} \tag{2.1a}$$

$$p_1 - p_2 = \rho\big\{(V_2^2 - (\alpha v^2 + (1-\alpha)V_1^2)\big\}, \qquad \text{mom. = 20} \tag{2.1b}$$

$\alpha = a/A$

By substituting (2.1a) into (2.1b),

$$\begin{aligned}\frac{p_1 - p_2}{\rho} &= \big((1-\alpha)V_1 + \alpha v\big)^2 - \alpha v^2 - (1-\alpha)V_1^2,\\ &= (1-\alpha)^2V_1^2 + 2\alpha(1-\alpha)vV_1 + \alpha^2v^2 - \alpha v^2 - (1-\alpha)V_1^2,\\ &= -\alpha(1-\alpha)V_1^2 + 2\alpha(1-\alpha)vV_1 + \alpha(1-\alpha)v^2,\\ &= -\alpha(1-\alpha)(v - V_1)^2,\end{aligned} \tag{2.2}$$

20 if arrived to properly

which is equivalent to the result given.

40 (b) By setting the shaft power, and the storage term to zero, then evaluating the flux terms in the mechanical energy balance,

$$\rho V_2 A\Big(\frac{1}{2}V_2^2 + \frac{p_2}{\rho}\Big) - \Big\{\rho va\Big(\frac{1}{2}v^2 + \frac{p_1}{\rho}\Big) + \rho(A - a)V_1\Big(\frac{1}{2}V_1^2 + \frac{p_1}{\rho}\Big)\Big\} = -P.L. \qquad 20$$

energy balance

By solving for the power loss

$$
\begin{aligned}
P.L. &= \left\{\rho v a\left(\frac{1}{2}v^2 + \frac{p_1}{\rho}\right) + \rho(A - a)V_1\left(\frac{1}{2}V_1^2 + \frac{p_1}{\rho}\right)\right\} - \rho V_2 A\left(\frac{1}{2}V_2^2 + \frac{p_2}{\rho}\right) \\
&= V_2 A(p_1 - p_2) + \frac{1}{2}\rho A\left\{\alpha v^3 + (1-\alpha)V_1^3 - V_2^3\right\}, && \text{(collected terms in } p_1 \text{ then used (2.1a)} \\
&= V_2 A(p_1 - p_2) + \frac{1}{2}\rho A\left\{\alpha v^3 + (1-\alpha)V_1^3 - (\alpha v + [1-\alpha]V_1)^3\right\} && \text{(substituted for } V_2 \text{ in flux terms)} \\
&= V_2 A(p_1 + p_2) + \frac{1}{2}\rho A\alpha(1-\alpha)(v - V_1)^2\{(1+\alpha)v + (2-\alpha)V_1\} && \text{(by identity given)} \\
&= V_2 A(p_1 - p_2) - \frac{1}{2}A(p_1 - p_2)\{(1+\alpha)v + (2-\alpha)V_1\} && \text{(by result of part (a))} \\
&= A(p_1 - p_2)\left\{(\alpha v + (1 - \alpha V_1) - \frac{1}{2}\{(1+\alpha)v + (2-\alpha)V_1\}\right\} && \text{(substituted for} V_2\text{)} \\
&= \frac{1}{2}A(p_1 - p_2)\{(\alpha - 1)v - \alpha V_1\},
\end{aligned}
$$

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partial credit for partially correct algebra

(2.3)

as claimed.

2 mean: 62
std: 29
median: 60