Ö. Savaş

NAME: **SOLUTIONS**

9:40-11:00 Thursday, October 9, 2014

ME-163 ENGINEERING AERODYNAMICS
MIDTERM EXAM

closed book, closed computer, open notes – no internal or external communication

1. (15+10) A typical semi-truck/trailer has a frontal area of 2.5 m (width) × 4.0 m (height) = 10 m^2. Their *typical* highway speed is 80 mph (35.8 m/s; yes, above the posted speed limit). Assume that the truck can be treated as a *bluff body*.

(a) Estimate the aerodynamic drag force on the truck.

(b) Estimate the engine power needed to overcome this drag force. For reference, these trucks have engines in the range of 225- 500 kW (300-650 hp, 1 kW=1.341 hp).

(a) $D = C_D \frac{1}{2} \rho U^2 A \approx 1.0 \times \frac{1}{2} \times 1.2 \times 35.8^2 \times 10 = 7700\ N$

(b) $\dot{W} = DU = 7700 \times 35.8 = 275\ kW = 370\ hp$

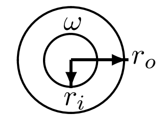
---

grading table

1 (25)	**2** (25)	**3** (25)	**4** (45)	**out of** (100)

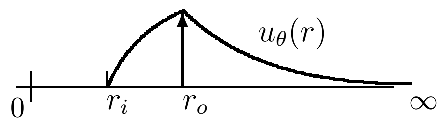
.

2. (25) Determine and sketch the velocity profile of a circular strip (washer) of uniform vorticity of inner and outer radii r_i and r_o. (Hint: Consider the circulation theorem in this axisymmetric configuration over in the three regions.)



$$\Gamma = \int_A \omega dA \Longrightarrow \int \omega(r)\ 2\pi r\ dr = u_\theta 2\pi r \Longleftarrow \int_{\mathcal{C}} \mathbf{u}\cdot d\mathbf{l} \quad \longrightarrow \quad u_\theta = \frac{\int \omega(r)\ r\ dr}{r}$$

- $r < r_i$: $u_\theta = 0$
- $r_i < r < r_o$: $u_\theta = \frac{1}{2}\omega(r - r_i^2/r)$
- $r_o < r$: $u_\theta = \frac{1}{2}\omega(r_o^2 - r_i^2)/r$



3. (25) A passenger airliner, flying at $M = 0.8$ at about an altitude of 11 km, crashed into the Atlantic Ocean in June 2009 after a series of events that were triggered by icing of the Pitot Tube speed indicator. At the beginning of the catastrophe, the tube indicated a speed of 270 knots which went down even further. What would have been the stagnation pressure reading at this speed of 270 knots? What would have been the correct pressure reading at the Pitot tube total head? (1 knot=0.514 m/s)

---

at 11 km, from Table 3, $T_\infty = 217K$, $p_\infty = 2.263 \times 10^4 P$, $\rho_\infty = 0.364 kg/m^3$, $a_\infty = \sqrt{\gamma RT} = 295 m/s$

$$u_\infty^2 = \frac{2\gamma}{\gamma - 1}\frac{p_\infty}{\rho_\infty}\left[\left(\frac{p_o}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \Longrightarrow \frac{p_o}{p_\infty} = \left[\frac{\gamma - 1}{2\gamma}\frac{\rho_\infty}{p_\infty}u_\infty^2 + 1\right]^{\frac{\gamma}{\gamma-1}} = \left[\frac{1}{7}\frac{\rho_\infty}{p_\infty}u_\infty^2 + 1\right]^{3.5}_{\gamma=1.4}$$

If 270 knots were true $u_\infty = 270 \times 0.514 = 139 m/s$

$$\frac{p_o}{p_\infty} = 1.164 \Longrightarrow p_o = 2.63 \times 10^4 P$$

At $M = 0.8$, the true speed, $u_\infty = a_\infty M = 236 m/s$

$$\frac{p_o}{p_\infty} = 1.524 = \frac{1}{0.656} \Longrightarrow p_o = 3.45 \times 10^4 P$$

Or, equivalently,, from Table 6, at $M = 0.8$, $p_\infty/p_o = 0.656 = 1/1.524$

---

4. (7×5+10=45) Consider the two-dimensional (r, θ) unsteady flow field given by the stream function

$$\psi(r, \theta, t) = \frac{1}{2} \ln \left[(r^2 + 1)^2 - 4r^2 \cos^2(\theta - t) \right]$$

(a) determine the velocity vector (u_r, u_θ) as $r \to \infty$.

(b) determine the stagnation points, if any.

(c) sketch the streamlines as $r \to \infty$.

(d) show that streamline pattern rotates at a constant angular speed without changing.

(e) determine the approximate form of streamlines as $r \to 0$.

(f) can you comment on the approximate form of streamlines as $(r, \theta) \to (\pm 1, t)$.

(g) can you identify the elements of the flow field.

(h) can you sketch the complete streamline pattern.

Hints:

$$\frac{d}{dx} \ln x = \frac{1}{x} \qquad \ln(1 + x) \sim x - \frac{x^2}{2} + \frac{x^3}{3} ... \qquad \frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$$

You may find it helpful to look at the flow pattern from the rotating reference frame by considering the pattern at $\theta = t$.

If you prefer to work with the corresponding potential function, it is

$$\varphi(r, \theta, t) = \tan^{-1} \left(\frac{r^2 \sin 2\theta - \sin 2t}{r^2 \cos 2\theta - \cos 2t} \right).$$

or, in complex form,

$$F(z) = \varphi + i\psi = i \ln(z^2 - e^{i2t})$$

$$(u_r, u_\theta) = \left(\frac{1}{r} \frac{\partial \psi}{\partial \theta}, -\frac{\partial \psi}{\partial r} \right) = \frac{4r}{2\left[(r^2 + 1)^2 - 4r^2 \cos^2(\theta - t) \right]} \Big(\sin 2(\theta - t),\ r^2 - \cos 2(\theta - t) \Big)$$

(a) $\psi \to 2 \ln r$, $(u_r, u_\theta) \to (0, 2/r)$, point vortex at origin with $\Gamma = 4\pi$.

(b) stagnation point at $r = 0$.

(c) as $r \to \infty$, streamlines become circular.

(d) $\theta - t = cons$ removes θ dependence.

(e) $\psi \to 2r^2 \cos 2(\theta - t)$, rotating stagnation point flow.

(f) tight circles.

(g) vortex pair of equal strengths

(h) see figure.

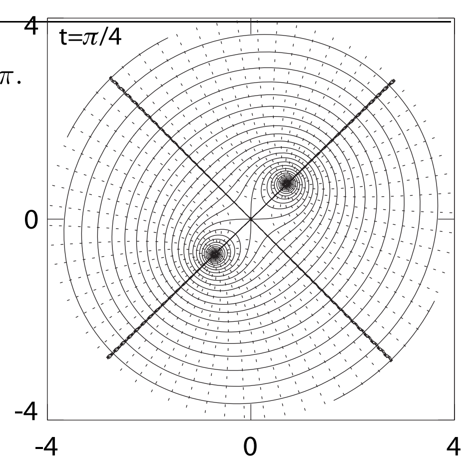


Figure 1: Streamlines (solid) and potential lines (dashed) of an orbiting vortex pair. Lines are drawn at $t = \pi/4$.