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9:40-11:00, Thursday, October 4, 2018

ME-163 ENGINEERING AERODYNAMICS
MIDTERM EXAM
closed book, open notes – no internal or external communication

1. Consider the flow field whose potential function is given as (in dimensionless from)

$$\varphi = x + \frac{1}{2}\,\frac{\sinh x}{\cosh x - \cos y}$$

   (a) Determine the velocity vector field (u, v).
   (b) Show that the normal velocity $v = 0$ on $y = \pm\pi$, hence, it represents a flow between two parallel walls.
   (c) Show that the flow is uniform at $x = \mp\infty$ between the walls.
   (d) Determine the stagnation points. *Note that the flow is symmetric with respect to the x-axis.*
   (e) Sketch the flow pattern.

$$(u, v) = (\varphi_x, \varphi_y) = \left(1 - \frac{1}{2}\frac{\cosh x\ \cos y - 1}{(\cosh x - \cos y)^2}, -\frac{1}{2}\frac{\sinh x\ \sin y}{(\cosh x - \cos y)^2}\right)$$

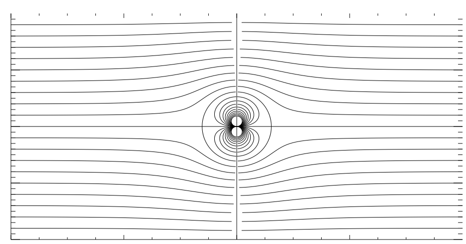
$\sin \pm\pi = 0$, hence, $v(x, y = \pm\pi) \equiv 0$. Therefore, $y = \pm\pi$ lines are streamlines which may be replaced with solid boundaries.

$\cosh x \to \infty$ as x $\to \mp\infty$, $\varphi \to x$, hence $\mathbf{u} \to (1, 0)$: uniform flow.

The x-axis ($y = 0$) is a streamline, since $v = 0$ along it. Along the x-axis,

$$u = 1 - \frac{1}{2(\cosh x - 1)}$$

$u = 0$ at $\cosh x = 3/2$, or at $x = \ln[(3 \mp \sqrt{5})/2] = \mp 0.962$. Stagnation points are at $(x, y) = (\mp 0.962, 0)$.



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flow past an oval in a channel: almost a circle

$$\text{aspect ratio} \quad \frac{a_x}{a_y} = \frac{\ln[(3 + \sqrt{5})/2]}{[2a_y \tan(a_y/2) = 1]} = \frac{0.9624}{0.9602} = 1.0023!$$

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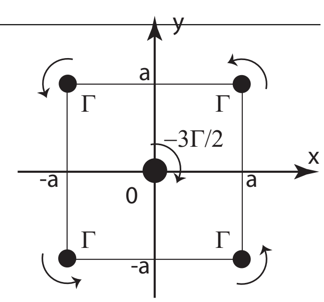
2. Consider four equi-strength point vortices of strength Γ at the vertices of a square and a vortex of strength $-3\Gamma/2$ at the center of the square. Describe the motion of this five-vortex system. Provide mathematical arguments to support your answer.



The system is stationary. At the corner vortices:

$$u_\theta = \frac{\Gamma}{2\pi}\left(2 \times \frac{cos45^\circ}{2a} + \frac{1}{2\sqrt{2}\,a}\right) - \frac{3\Gamma/2}{2\pi\sqrt{2}\,a} = 0$$

Similarly, $u_r = 0$, therefore the corner vortices to not move. By symmetry, the induced velocity at the central vortex by the four corner vorticies also vanishes. Hence, the whole system stays put.

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3. Solar Impulse, the first solar power airplane to circumnavigate the earth, has a mass of about 2300 kg, wing span of 72 meters, and average chord of 3.6 meters. Its take-off speed is 10 m/s, and design cruise speed is 25 m/s at 8500 meter altitude.

   Calculate the lift coefficients (a) during take off and (b) during cruising

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A} = \frac{mg}{\frac{1}{2}\rho U^2 bc}$$

(a) Take off: $C_L \sim 1.41$ at 10 m/s at sea level $\rho = 1.23\ kg/m^3$

(b) Cruise: $C_L \sim 0.55$ at 25 m/s, at 8500 meter $\rho = 0.5\ kg/m^3$

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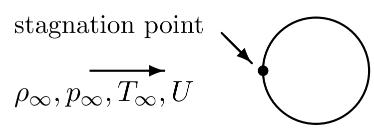
4. Consider a sphere moving in air at $U = 150$ m/s at sea level in standard atmosphere. Determine the pressure, density and temperature at the front stagnation point. How does the density compare to that at far upstream?



Compressible flow calculation since M=150/340=0.441 is significantly high:

$$\frac{\gamma}{\gamma-1}\frac{p_\infty}{\rho_\infty} + \frac{1}{2}U^2 = \frac{\gamma}{\gamma-1}\frac{p_o}{\rho_o} \quad \text{and} \quad \frac{p_\infty}{\rho_\infty^\gamma} = \frac{p_o}{\rho_o^\gamma} \quad \Longrightarrow \quad p_o = p_\infty\left[1 + \frac{\gamma-1}{2\gamma}\frac{\rho_\infty U^2}{p_\infty}\right]^{\frac{\gamma}{\gamma-1}} = p_\infty\left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}$$

put in numbers $(p_\infty, \rho_\infty, T_\infty) = (1.01325 \times 10^5, 1.225, 288.15)$[SI]

M Kernel=1.03893

$$p_o = 1.01325 \times 10^5 \times \left[1 + \frac{1.4-1}{2 \times 1.4}\frac{1.225 \times 150^2}{1.01325 \times 10^5}\right]^{\frac{1.4}{1.4-1}} = 1.01325 \times 10^5 \times 1.1430\ Pa = 1.1581 \times 10^5\ Pa$$

$\rho_o = \rho_\infty(p_o/p_\infty)^{1/\gamma} = 1.225 \times (1.1581 \times 10^5/1.01325 \times 10^5)^{1/1.4} = 1.225 \times 1.100 = 1.348$ kg/m^3

$T_o = p_o/(\rho_o R) = 1.1581 \times 10^5/(1.348 \times 8.314/0.02897) = 299\ K$

$(p_o, \rho_o, T_o) = (1.1581 \times 10^5, 1.348, 299)$[SI]