NE 180
Midterm II
Fall Semester 2001
Solutions

November 20, 2001

1. a.

$$n\tau_i = \frac{2.09 \times 10^7 (m_i/m_p)^{1/2} T_i^{3/2}}{\ln \Lambda_{ii}}$$

$$\tau_i = 31ms$$

$$\omega_{ci}\tau_i = 4.75 \times 10^6$$

$$q_i = -\kappa_\perp^i \nabla T_i$$

$$\kappa_\perp^i = 2.0 \frac{nkT_i\tau_i}{m_i(\omega_{ci}\tau_i)^2} = 1.97 \times 10^{17}\ m^{-1}s^{-1}$$

$$\nabla T_i = 15000 \times 1.6 \times 10^{-19} = 2.4 \times 10^{-15}\ Jm^{-1}$$

$$\boxed{q_i = 473\ \mathrm{W\ m^{-2}}}$$

1. b.

$$Q_{ie} = 3\frac{m_e}{m_i}\frac{n(T_e - T_i)}{\tau_e}$$

$$n\tau_e = \frac{3.44 \times 10^5 T_i (eV)^{3/2}}{\ln \Lambda} \text{ (cgs units)}$$
$$= 1.965 \times 10^{10} \text{cm}^{-3}\text{s}$$
$$= 1.965 \times 10^{16} \text{m}^{-3}\text{s}$$
$$\tau_e = 196.5 \mu\text{s}$$

$$\boxed{Q_{ie} = 333 \text{ kW m}^{-3}}$$

1. c.

$$\vec{J} = \frac{\vec{B} \times \vec{\nabla} p}{B^2} = -\hat{y}\frac{(10^{20})(1.6 \times 10^{-19})(15000)}{4}$$

$$\boxed{= -\hat{y}\ 60000 Am^{-2}}$$

1. d.

Plasma resistivity $= \eta = 2.8 \times 10^{-8} T_e (keV)^{-3/2} = 8.85 \times 10^{-10} \Omega - \text{m}$

$$\boxed{P_\Omega = \eta j^2 = 3.18 \times 10^{-6} \text{ MW m}^{-3}}$$

2. a.

$$n_e = \frac{\rho}{M} N_A = \frac{25}{(2.5)(1.67 \times 10^{-24})(6.02 \times 10^{23})} = 6.02 \times 10^{24} \text{ cm}^{-3}$$

$$f_{pe} = 89 \times 10^9 (n_e/10^{14})^{1/2} = 2.18 \times 10^{16} \text{ Hz}$$

$$\boxed{\omega_{pe} = 1.37 \times 10^{17} \text{ s}^{-1}}$$

2. b.

$$f_B = 0.5$$

$$M = (4\pi/3)\rho R^3 = 6.7\ g$$

$$\boxed{Y = Y' M f_B = 3.39 \times 10^{11}\ \text{J}}$$

2. c.

$$\lambda_{crit} = \frac{c}{f_{pe}}$$

$$\lambda_{crit}(\mathring{A}) = \frac{3 \times 10^{10}(10^8)}{2.18 \times 10^{16}} = 137\ \mathring{A}$$

$$\boxed{E^{\gamma}_{crit} = 12400/137 = 90.25\ \text{eV}}$$

2. d.

$$\omega^2 = \omega_{pe}^2 + k^2 c^2$$

$$v_\phi = \omega/k = 2c \rightarrow \omega^2 = 4k^2c^2$$

Or

$$\omega_p^2 = 3k^2c^2 = 3/4\omega^2$$

And then $n = (3/4)^{1/2} n_{crit}$ for the laser frequency.

$$f_{laser} = 3 \times 10^{18}/350 = 8.57 \times 10^{15}\ \text{Hz}$$

so that $n_{crit} = (f_{laser}/8.9 \times 10^{10})^2 \times 10^{14} = 8.03 \times 10^{23}\ \text{cm}^{-3}$ And thus

$$\boxed{n = (3/4)^{1/2} n_{crit} = 13.37\% \text{ of compressed density}}$$