Solution set for Mid-term I

1.(a) $\frac{dN_{49}(t)}{dt} = N_{28}^{o}\sigma_{28}\phi_M - N_{49}(t)\sigma_{49}\phi_M,\ t>0,$ subject to $N_{49}(0) = N_{49}^{o}.$

Let $N_{49}(t) = N_{49,1}(t) + N_{49,2}(t)$. For each function, the following sub-problems are set:

$$\frac{dN_{49,1}(t)}{dt} = N_{28}^{o}\sigma_{28}\phi_M - N_{49}(t)\sigma_{49,1}\phi_M,\ t>0,\ \text{subject to}\ N_{49,1}(0)=0, \text{and}$$

$$\frac{dN_{49,2}(t)}{dt} = -N_{49}(t)\sigma_{49}\phi_M,\ t>0,\ \text{subject to}\ N_{49,2}(0)=N_{49}^{o}.$$

For $N_{49,1}(t)$, the solution is $N_{49,1}(t) = \frac{\sigma_{28}N_{28}^{o}}{\sigma_{49}}\{1-\exp(-\sigma_{49}\phi_M t)\}, t\geq 0,$ and

for $N_{49,2}(t)$, the solution is $N_{49,2}(t) = N_{49}^{o}\exp(-\sigma_{49}\phi_M t), t\geq 0.$

(b) The burn-up is defined as $B = 9.5\times10^5 w = 9.5\times10^5 \frac{239N_F(49)}{239N_{49}^{o}+238N_{28}^{o}}$.

The number of fission pairs due to fissions in Pu-239 is obtained by integrating $N_{49}(t)$ as

$$N_F(49) = \phi_M\sigma_{49}^{f}\int_0^t N_{49}(t')dt' \ \text{where}\ N_{49}(t) = N_{49}^{o}\exp(-\sigma_{49}\phi_M t) + \frac{\sigma_{28}N_{28}^{o}}{\sigma_{49}}\{1-\exp(-\sigma_{49}\phi_M t)\}, t\geq 0 .$$

(c) $N_{49}(t) \leq N_{49}^{o}$. Substituting the solution obtained in (a) yields $N_{49}^{o} > N_{28}^{o}\,\sigma_{28}/\sigma_{49}$.

2. Denote the natural uranium mass per year as N, the depleted uranium mass per year as W, and the mass of enriched fuel as F.

(a) $0.2W + 3.3F = 0.711N,\ F + W = N$, where F = 27.2 MT. Thus, N = 165 MT.

(b) The fuel mass F is obtained by $\frac{0.95\times1000}{0.325}\times\frac{1000}{33,000} = 88.6$ kg/day = 32.3 MT/year . The balance equations are: $0.3W + 3.3F = 0.711N,\ F + W = N$. Thus, N = 236 MT

(c) The fuel mass F is obtained by $\frac{0.8\times1000}{0.34}\times\frac{1000}{33,000} = 71.3$ kg/day = 26.0 MT/year . The balance equations are: $0.3W + 3.3F = 0.711N,\ F + W = N$ Thus, N = 190 MT

3. (a) Mass of uranium contained in 1 metric ton of uranium ore is 10 kg. In natural uranium, 0.71% of U-235 and 99.29% of U-238 are contained. Thus, there are 9.929 kg of U-238 and 0.071 kg of U-235. For U-238, radioactivity is 1.22E8 Bq, or 3.31E-3 Ci. For U-235, 5.63E6 Bq, or 1.52E-4 Ci. Because these uranium isotopes are in secular equilibria with daughters in the ore, the total radioactivity of the ore is obtained by taking into account radioactivity of these daughters. For U-238, there are 13 radioactive daughters, whereas for U-235, there are 11 radioactive daughters. Thus, 3.31E-3 x 14 + 1.52E-4 x 12 = 4.81E-2 Ci or 1.78E9 Bq

(b) From Uranium mill, the product is yellow cake, containing U_3O_8, Ra (depending on separation process applied at the mill), and non-radioactive impurities such as vanadium. The waste effluents are: (1) airborne Rn, and (2) tailings containing all decay daughters of uranium isotopes, 5 to 10 % of uranium originally contained in the ore, and crashed rock.

From Conversion, the product is UF_6 in gaseous form. Wastes are: (1) Ra that has escaped from the mill and other non-radioactive impurities, and (2) uranium-contaminated wastes during operation, which is < 0.5% of U.

From isotope separation, the product is enriched uranium, containing 3 to 4 % U-235. The waste streams are (1) depleted uranium containing 0.3% U-235 and U-contaminated wastes generated from operation.

4. (a) $K_H = \frac{[HNO_3\cdot TBP]_{org}}{[H^+]_{aq}[NO_3^-]_{aq}[TBP]_{org}}$

(b) $D_H = \dfrac{[HNO_3 \cdot TBP]_{org}}{[H^+]_{aq}}$

(c) $D_H = C \Big/ \left[\left[H^+ \right] + \dfrac{1}{K_H \left[NO_3^- \right]} \right]$

Thus, if pH increases, $[H^+]$ decreases, and D_H increases.

5.

(a) Carbon-14 is generated by activation of ${}^{14}_{7}N$ and ${}^{17}_{8}O$ both included in fuel and coolant water. Nitrogen is included in the pores in fuel pellets because sintering of UO2 powder into pellets is made under N2 atmosphere to control oxidation states of uranium. Nitrogen is also dissolved in the coolant water. Oxygen is included in UO2 and water. Carbon-14 has a long-half life (5700 years), and in a gaseous phase in the spent fuel. It leaks from failed waste packages in the final repository eventually into the atmosphere.

(b) I-129: Long-lived fission product isotope, and one of major contributors for long-term environmental impact. Sr-90 or Cs-137: major heat-emitting fission product isotopes. They determine the magnitude of heat emission and radioactivity from spent fuel for the first centuries. Pu-239: generated as neutron absorption in U-238. This is also a thermally fissile isotope. Because of this isotope, we need to make careful considerations for criticality safety in various stages of waste management. Np-237: generated by successive neutron absorption in U-235 and beta decay. Long half life (2 million years). Major contributor for the environmental impact of the Yucca mountain repository. Could be used as fuel material in fast reactors.

(c) (c-1) False: Some nuclides, such as Pu isotopes, increase their radioactivity due to decay of their precursors. See Figures in Page 25.

(c-2) False: Decommissioning wastes contain activated materials, such as Co-60. Because of these nuclides, some portion of decommissioning wastes must be treated as radioactive wastes.

(c-3) False: As far as we rely on chemical equilibrium for separation, perfect separation is not possible.

(c-4) False: Many fission product isotopes have short half lives and decay to stable species. See Table in Page 22.

(c-5) False: There are radionuclides with half lives longer than 10 years in activated materials.