Name: ____________________

Student ID number: ____________________

# Midterm 1

Statistics 153 Introduction to Time Series

March 7th, 2019

**General comments:**

1. Flip this page only after the midterm has started.
2. Before handing in, write your name one **every** sheet of paper!
3. Anyone caught cheating on this midterm will receive a failing grade and will also be reported to the University Office of Student Conduct. In order to guarantee that you are not suspected of cheating, please keep your eyes on your own materials and do not converse with others during the midterm.

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1. Consider the following model for time series data $X_t = X_{t-1} + Z_t + \delta$, where $\delta$ is some non-zero constant and $Z_t$ is white noise with variance $\sigma^2$.

   (a) Give the definition of weak and strong stationarity.

   (4 Points)

   (b) Show that there exist no stationary solution for $X_t$ in the above model.

   (2 Points)

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(c) From now on suppose that $X_0 = 0$. Compute the mean and the variance of $X_t$ for all $t > 0$.

(3 Points)

(d) Is $X_t$ homoscedastic? Explain.

(1 Points)

(e) Propose an invertible function $f(\cdot)$ such that the transformed data $f(X_t)$ has approximately constant variance. Explain.

*Hint: You may assume that all your observations are positive.*

(3 Points)

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(f) Propose an invertible transformation of $X_t$ such that it is stationary. Explain.

(3 Points)

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2. Consider the stationary, zero-mean AR(1) model $X_t = 0.5X_{t-1} + Z_t$ and the MA(1) model $W_t = 0.5Z_{t-1} + Z_t$, where $Z_t$ is some white noise with variance $\sigma^2$.

   (a) For each of $Z_t, W_t$, and $X_t$ give the ACVF and ACF function.

      i. For $Z_t$:

      (1 Points)

      ii. For $W_t$:

      (2 Points)

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iii. For $X_t$:

(2 Points)

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(b) For each of $Z_t, W_t$, and $X_t$ give the approximate mean and variance of its sample ACF at lag 2 for $n = 100$ observations.

*Hint: Recall Bartlett's formula* $W_{ij} =$
$\sum_{m=1}^{\infty} \left(\rho(m+i) + \rho(m-i) - 2\rho(i)\rho(m)\right)\left(\rho(m+j) + \rho(m-j) - 2\rho(j)\rho(m)\right)$

i. For $Z_t$:

(2 Points)

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ii. For $W_t$:

(4 Points)

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iii. For $X_t$:

(4 Points)

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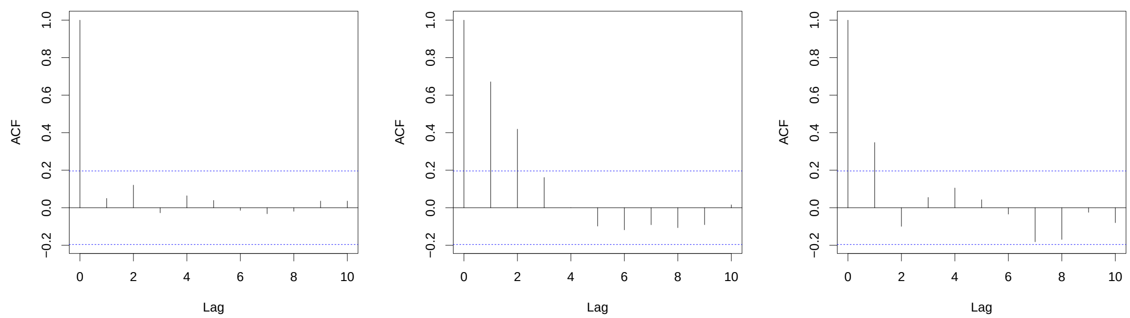


Figure 1: Sample ACFs of different time series data.

(c) Figure 1 shows sample ACFs for each of the three models for $n = 100$ observations. Which figure corresponds to which process? Explain.

(3 Points)

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3. For zero mean time series data $\{X_t\}$ consider the model $(1 - 0.2B)(X_t - 0.5X_{t-1}) = (Z_t - 0.6Z_{t-1} + 0.05Z_{t-2})$, where $\{Z_t\}$ is white noise with variance $\sigma^2 = 4$.

   (a) Identify $\{X_t\}$ as an ARMA(p,q) model and give its MA and AR polynomials.

   (4 Points)

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(b) Is the model invertible and causal?

(2 Points)

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(c) Find its unique stationary solution.

(4 Points)

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(d) Compute its ACVF.

(4 Points)

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(e) Assume someone wants to use this model to predict weekly car sales. On average the company sells 100 cars per week. Two weeks ago they sold 95 cars and last week they sold 101 cars. Based on this, what is the best linear predictor of car sales next week?

*Hint: You do not have to compute the actual value, it is enough to write down a linear system of equations that needs to be solved.*

(3 Points)

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4. A scientist considers the model $X_t = m_t + s_t + W_t$ for some time series data, where $m_t = a\,t + b$ is a linear trend function with parameters $a, b$ and $s_t$ is a seasonal component with period 2, that is, $s_t = s_{t+2}$ for all $t$. $W_t$ is some zero mean stationary process.

   (a) First, the scientist wants to estimate the trend function $m_t$ using a filter of the form $1 + \alpha B + \beta B^2 + \gamma B^3$, where $B$ denotes the backshift operator and $\alpha, \beta, \gamma$ are parameters. How should she chose the parameters $\alpha, \beta, \gamma$ such that the filtered time series is an unbiased estimator of the trend $m_t$, that is, $E((1 + \alpha B + \beta B^2 + \gamma B^3)X_t) = m_t$?
   *Hint: First, argue that without loss of generality you can assume that* $s_1 + s_2 = 0$.
   (5 Points)

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(b) Is $X_t$ a stationary process? Explain.

(1 Points)

(c) Propose a transformation using differencing to make the process stationary. Explain.

(3 Points)

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(d) For the stationary process $W_t$ the scientist considers two different models:

- an MA(1) model,
- an AR(1) model.

For both of these choices identify the transformed data from (4c) as some ARMA model.

*Hint: It is enough to state the orders of the respective ARMA models with explanation.*

(6 Points)

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5. For each statement, indicate whether it is true or false and give a short explanation.
   *You only get points when both, True/False and the explanation, are correct.*

   (a) For the sample autocorrelations of $n = 1,000$ i.i.d. white noise random variables at lags $h = 1, \dots, 100$, you expect on average 5 of them to be larger than 1.96 in absolute value.

   [ ] True [ ] False

   Explanation: ______________________________________________

   ______________________________________________

   (b) The sample autocorrelations of an AR(1) process with i.i.d. white noise are (for large sample size) approximately i.i.d..

   [ ] True [ ] False

   Explanation: ______________________________________________

   ______________________________________________

   (c) Applying a linear (time invariant) filter to a stationary process results again in a stationary process.

   [ ] True [ ] False

   Explanation: ______________________________________________

   ______________________________________________

   (d) When you want to fit a seasonal parametric function of the form $s_t = a_0 + \sum_{f=1}^{k} (a_f \cos(2\pi ft/d) + b_f \sin(2\pi ft/d))$ with parameters $a_0, a_1, \dots, a_k, b_1, \dots, b_k$ it can be helpful to chose $k > d/2$.

   [ ] True [ ] False

   Explanation: ______________________________________________

   ______________________________________________

   (e) A time series $\{X_t\}$ where $X_t$ follows a Gaussian distribution for each $t$ is a Gaussian process.

   [ ] True [ ] False

   Explanation: ______________________________________________

   ______________________________________________

   (f) Whether a time series is invertible or not is fully determined by its finite dimensional distributions.

   [ ] True [ ] False

   Explanation: ______________________________________________

   ______________________________________________

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(g) Whether a time series is strongly stationary or not is fully determined by its mean and covariance function.

[ ] True [ ] False

Explanation: ________________________________________

____________________________________________________

(h) Whether a Gaussian process is strongly stationary or not is fully determined by its mean and covariance function.

[ ] True [ ] False

Explanation: ________________________________________

____________________________________________________

(8 Points)