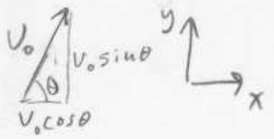
Mike DeWeese

1.) a.) constant $\vec{a}$: $\cancel{v_{top\,y}^2}^{\,0} = v_{oy}^2 + 2(y_{top} - y_o)a_y$ $\qquad g \equiv +10\,m/s^2$



$$0 = v_o^2 \sin^2\theta + 2(y_{top} - y_o)(-g)$$

$$y_{top} - y_o = \frac{v_o^2 \sin^2\theta}{2g}$$ ← height of hiest point above height at which ball was kicked.

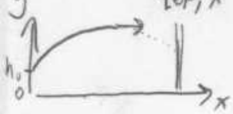
$$y_{top} - y_{ground} = \boxed{\frac{v_o^2 \sin^2\theta}{2g} + h_o}$$

b.) const. $\vec{a}$: $v_y = v_{oy} + a_y t_{top}$

$$0 = v_o \sin\theta - g t_{top}$$

$$t_{top} = \boxed{\frac{v_o \sin\theta}{g}}$$

c.) at highest point, $v_y = 0$. v_x is constant throughout trajectory:



$$v_{top,x} = v_{xo} = v_o \cos\theta$$

velocity of goal posts relative to ball at highest point $\equiv \vec{v}_{g.p.}'$

$$\vec{v}_{g.p.}' = \cancel{\vec{v}_{g.p.}}^{\,0} - \vec{v}_{ball} = \boxed{-v_o \cos\theta\,\hat{i}}$$

d.) time to reach $x = D$: $x = \cancel{x_o}^{\,0} + v_{ox} t + \frac{1}{2}\cancel{a_x}^{\,0} t^2$

$$D = v_o \cos\theta\, t \Rightarrow t = \frac{D}{v_o \cos\theta}$$

$v_{0,min} \equiv$ min v_o to pass over H at $x = D$:

const. a_y: $y = y_o + v_{yo} t + \frac{1}{2} a_y t^2$

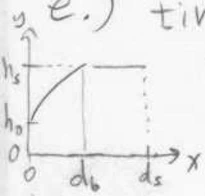
$$H = h_o + \cancel{v_{o\,min}} \sin\theta \frac{D}{\cancel{v_{o\,min}} \cos\theta} - \frac{1}{2} g \frac{D^2}{v_{min}^2 \cos^2\theta}$$

$$-(H - h_o) + D\tan\theta = \frac{g D^2}{2 v_{o\,min}^2 \cos^2\theta}$$

$$\frac{2\cos^2\theta}{g D^2} v_{o\,min}^2 = \frac{1}{D\tan\theta - (H - h_o)}$$

$$v_{o\,min} = \boxed{\frac{D}{\cos\theta}\sqrt{\frac{g}{2D\tan\theta - 2(H - h_o)}}}$$

e.) time for ball to reach height h_s:



const. a_y: $h_s = h_o + v_o \sin\theta\, t - \frac{1}{2} g t^2$ $\qquad at^2 + bt + c = 0 \Rightarrow t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic formula $\Big\}$ $t = \dfrac{+v_o \sin\theta \pm \sqrt{v_o^2 \sin^2\theta - 2g(h_s - h_o)}}{+g}$ $\qquad h_s = h_o \Rightarrow t = 0 \Rightarrow$ "−" in front of $\sqrt{\ }$ ...

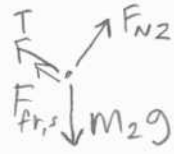
$v_{bx} = v_o \cos\theta$, $v_{sx} = -v_s$ both ball & stanford player have const. v_x

$$d_s = (|v_{sx}| + v_s)t = \boxed{(v_o \cos\theta + v_s)\frac{v_o \sin\theta - \sqrt{v_o^2 \sin^2\theta - 2g(h_s - h_o)}}{g}}$$

2.

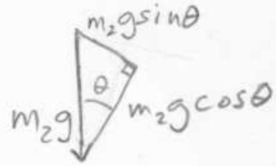
a.) FBD m_1

FBD m_2



b.) N2L m_1 x: $-F_{N1} + \cancel{F_{rope,x}}^{0} + \cancel{F_{fr,s,x,1}}^{0} + m_1 g \sin\theta = m_1 \cancel{a_{1x}}^{0}$

$$\boxed{F_{N1} = m_1 g \sin\theta}$$



c.) N2L m_1 y: $T - m_1 g \cos\theta = m_1 \cancel{a_{1y}}^{0 \Leftarrow \text{static}}$

$T = m_1 g \cos\theta$ — (1)

N2L m_2 x: $-T - F_{fr,s} + m_2 g \sin\theta = m_2 \cancel{a_{2x}}^{0 \Leftarrow \text{static}}$ — (2)

$F_{fr,s} = 0$ in (2) $\Rightarrow$ $T = m_2 g \sin\theta$ — (3)

(1) & (3) $\Rightarrow$ $m_1 g \cos\theta = m_2 g \sin\theta$

$\tan\theta = \frac{m_1}{m_2}$

$$\boxed{\theta = \tan^{-1}(m_1/m_2)}$$

d.) FBD m_2

N2L m_2 y: $F_{N2} - m_2 g \cos\theta = m_2 \cancel{a_{2y}}^{0}$ — (4)

N2L m_2 x: $-T + F_{fr,k} + m_2 g \sin\theta = m_2 a_{2x}$ — (5)

N2L m_1 y: $T - m_1 g \cos\theta = m_1 a_{1y}$ — (6)

constraint: $(y_p - y_1) + (x_2 - x_p) + \frac{\pi}{2} R_p = L$

radius of pulley — length of rope

$\dot{y}_p = \dot{x}_p = 0$

so: $-v_{1y} + v_{2x} = 0$

$a_{1y} = a_{2x}$ — (7)

kinetic friction: $F_{fr,k} = \mu_k F_{N2}$ — (8)

(4) & (5) & (8) $\Rightarrow$ $-T + \mu_k m_2 g \cos\theta + m_2 g \sin\theta = m_2 a_{2x}$ — (9)

(6), (7) & (9) $\Rightarrow$ $-T + m_2 g(\mu_k \cos\theta + \sin\theta) = \frac{m_2}{m_1}(T - m_1 g \cos\theta)$

$T\left(\frac{m_2}{m_1} + 1\right) = m_2 g(\mu_k \cos\theta + \sin\theta) + m_2 g \cos\theta$

$$\boxed{T = \frac{m_1 m_2}{m_1 + m_2} g\left((\mu_k + 1)\cos\theta + \sin\theta\right)}$$ — (10)

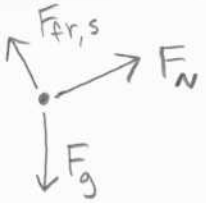
cont. →

2.e.) (10) in (6) with $a_{1y} = 0 \Rightarrow \cancel{m_1} \cancel{g} \cos\theta = \frac{\cancel{m_1} m_{2,max}}{m_1 + m_{2,max}} \cancel{g} \left[(\mu_k + 1)\cos\theta + \sin\theta \right]$

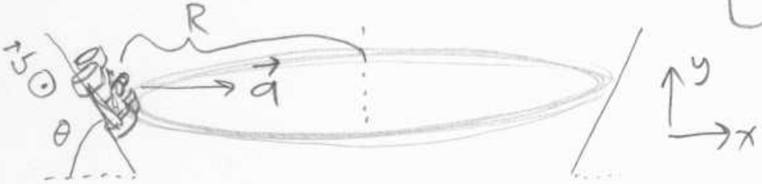
$$m_1 + \cancel{m_{2,max}} = m_{2,max} \left[(\mu_k + \cancel{1}) + \frac{\sin\theta}{\cos\theta} \right]$$

$$\boxed{m_{2,max} = \frac{m_1}{\mu_k + \tan\theta}}$$

3. a.) FBD car



b.) uniform circular motion: $\boxed{\vec{a} = \frac{v^2}{R}\hat{i}}$ to the <u>right</u> ✓①



c.) $\underset{car}{N2L}$ x: $-F_{fr,s}\cos\theta + F_N \sin\theta = m a_x \overset{①}{=} m\frac{v^2}{R}$ — ②

$\underset{car}{N2L}$ y: $F_{fr,s}\sin\theta + F_N\cos\theta - mg = m\cancel{a_y} \to 0$ moving in horizontal circle

$$F_N = \frac{mg}{\cos\theta} - F_{fr,s}\frac{\sin\theta}{\cos\theta} \quad — ③$$

③ in ② $\Rightarrow$ $-F_{fr,s}\cos\theta + mg\frac{\sin\theta}{\cos\theta} - F_{fr,s}\frac{\sin^2\theta}{\cos\theta} = m\frac{v^2}{R}$

$$F_{fr,s}\left(\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}\right) = -m\frac{v^2}{R} + mg\frac{\sin\theta}{\cos\theta}$$

$\cos^2\theta + \sin^2\theta = 1 \Rightarrow$ $F_{fr,s} = m\left(g\sin\theta - \frac{v^2}{R}\cos\theta\right)$ — ④

so: $\boxed{|F_{fr,s}| = m\left|g\sin\theta - \frac{v^2}{R}\cos\theta\right|}$

d.) $\theta = 45°$: $\sin(\theta) = \cos(\theta)$ in ④ $\Rightarrow$ $F_{fr,s} = \frac{m}{\sqrt{2}}\left(g - \frac{v^2}{R}\right)$ — ⑤

this eqn. assumes $F_{fr,s}$ points <u>up hill</u> as in my F.B.D above.

⑤ & $\vec{F}_{fr,s}$ downhill $\Rightarrow$ $\boxed{\frac{v^2}{R} > g}$

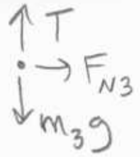
e.) no sliding if $v = 0$ & ② $\Rightarrow$ $-F_{fr,s}\frac{1}{\sqrt{2}} + F_N\frac{1}{\sqrt{2}} = m\cancel{\frac{v^2}{R}} \to 0$

$$F_N = F_{fr,s}$$

static friction: $F_{fr,s} \leq \mu_s F_N$

so $\boxed{\mu_s \geq 1}$ (big!)

4.) a.) F.B.D.: m_3



N2L m_3 y: $T - m_3 g = m_3 \cancel{a_{3y}} \to 0$ ⇐ m_3 not accel. up or down ✓

$$\boxed{T = m_3 g} \quad (1)$$

c.) constraint: rope has constant length $= L$

$$(x_p - x_2) + (y_p - y_3) + \frac{\pi}{2} R_p = L$$

$$v_{px} - v_{2x} + \cancel{v_{py}}^{\,0} - v_{3y} = 0$$

$$a_{px} - a_{2x} - a_{3y} = 0 \quad (5)$$

2 → 5
3 → 6
4 → 7
5 → 8

pulley mounted on M_1 ⇒ $x_1 = x_p + \text{constant}$

$$v_{1x} = v_{px}$$

$$a_{1x} = a_{px} \quad (6)$$

(5) & (6) ⇒ $a_{1x} - a_{2x} - a_{3y} = 0$ so: $a_{3y} = 0 \Rightarrow a_{1x} = a_{2x}$ (7)

finally: $x_1 = x_3 + \text{constant}$

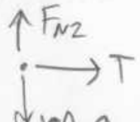
$$v_{1x} = v_{3x}$$

$$a_{1x} = a_{3x} \quad (8)$$

(7) & (8) $a_{1x} = a_{2x} = a_{3x}$ ⇒ N2L m_1 & m_2 & m_3 x: $F = (m_1 + m_2 + m_3) a_{2x}$

(4) ⇒ $\boxed{F = (m_1 + m_2 + m_3) g \frac{m_3}{m_2}}$

b.) FBD m_2



N2L m_2 x: $T = m_2 a_{2x}$ (2)

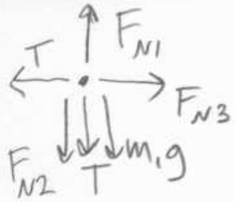
$$a_{2y} = 0 \quad (3)$$

(1) & (2) ⇒ $m_3 g = m_2 a_{2x}$

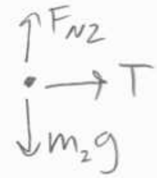
$$a_{2x} = g \frac{m_3}{m_2} \quad (4)$$

(3) & (4) ⇒ $\boxed{|\vec{a}_2| = g \frac{m_3}{m_2}}$

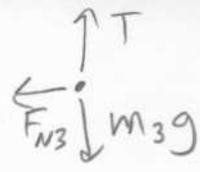
4. d.) F.B.D. m_1



F.B.D. m_2



F.B.D. m_3



N2L m_1 x: $F_{N3} - T = m_1 a_{1x}$ — (9) ✓

N2L m_2 x: $T = m_2 a_{2x}$ — (10) ✓

N2L m_3 x: $-F_{N3} = m_3 a_{3x}$ — (11) ✓

N2L m_3 y: $T - m_3 g = m_3 a_{3y}$ — (12) ✓

constraints $\begin{cases} (5) \;\&\; (6) \Rightarrow a_{1x} - a_{2x} - a_{3y} = 0 & \text{— (13) ✓} \\ a_{1x} = a_{3x} & \text{— (14) ✓} \end{cases}$

(14) & (9) & (11) $\Rightarrow$ $-m_3 a_{1x} - T = m_1 a_{1x} \Rightarrow a_{1x} = \frac{-T}{m_1 + m_3}$ — (15) ✓

(12) & (13) $\Rightarrow$ $T - m_3 g = m_3 (a_{1x} - a_{2x})$ — (16) ✓

(10) & (15) $\Rightarrow$ $a_{1x} = -a_{2x} \frac{m_2}{m_1 + m_3}$ — (17)

(10) & (16) $\Rightarrow$ $m_2 a_{2x} - m_3 g = m_3 (a_{1x} - a_{2x})$ — (18)

(17) & (18) $-(m_1 + m_3) a_{1x} - m_3 g = m_3 \left(a_{1x} + \frac{m_1 + m_3}{m_2} a_{1x}\right)$

$$a_{1x}\left(-m_1 - m_3 - m_3 - \frac{m_3}{m_2}(m_1 + m_3)\right) = m_3 g$$

$$a_{1x}\left(-m_1 m_2 - 2 m_2 m_3 - m_3 m_1 - m_3^2\right) = m_2 m_3 g$$

$$\boxed{a_{1x} = -\frac{m_2 m_3 \, g}{m_1(m_2 + m_3) + m_3(2 m_2 + m_3)}} \quad \text{— (19)}$$

e.) (15) & (19) $\Rightarrow$ $\boxed{T = \frac{m_2 m_3 (m_1 + m_3) g}{m_1(m_2 + m_3) + m_3(2 m_2 + m_3)}}$