1. (10 + 10 points)

(a) Calculate the binding energy, in MeV, per nucleon in $^{30}_{15}P$.

Ans $\Delta E = \Delta m c^2$ $\Delta m = m[^{30}_{15}P] - 15m[^1_1H] - 15m[^1_0n]$

$\Delta m = (29.9783\ u) - 15(1.0078\ u) - 15(1.0087\ u)$

$\Delta m = -0.2692\ u$

$\Delta E = \left(-.2692\ u \cdot \frac{1.66 \times 10^{-27} kg}{1\ u}\right)\left(2.99 \times 10^8 \frac{m}{s}\right)^2$

$\Delta E = -4.00 \times 10^{-11}\ J \cdot \frac{1\ MeV}{1.602 \times 10^{-13} J} = -249.4\ MeV$

$\frac{\Delta E}{\#\ of\ nucleons}$ = binding energy $\frac{249.4\ MeV}{30\ nucl.} = \boxed{8.31\ MeV/nucleon}$

(b) The radioactive decay of $^{30}_{15}P$ occurs through positron emission. Write the balanced nuclear decay reaction for this process and calculate the maximum kinetic energy, in MeV, carried off by the positron.

$\boxed{^{30}_{15}P \longrightarrow {}^{0}_{1}e^{+} + {}^{30}_{14}Si + \nu}$

$\Delta E = \Delta m c^2$ $\Delta m = (-m[^{30}_{15}P]) + 2m[^0_1e^+] + m[^{30}_{14}Si]$

$\Delta m = (-29.9783\ u) + 2(.00055\ u) + (29.9738\ u)$

$\Delta m = -.0034\ u$

$\Delta E = \left(-.0034\ u \cdot 1.66 \times 10^{-27} \frac{kg}{u}\right)\left(2.99 \times 10^8 \frac{m}{s}\right)^2$

$\Delta E = -5.05 \times 10^{-13}\ J \cdot \frac{1\ MeV}{1.602 \times 10^{-13}} = -3.15\ MeV$

Change in Energy equals KE of ejected positron so

$\boxed{KE_{max} = 3.15\ MeV \text{ per positron}}$

2. (10 + 10 points) The half-life of ^{14}C is 5730 years and 1.00 g of modern wood charcoal has an activity of 0.255 Bq. *Assume charcoal is pure carbon.*

(a) Calculate the number of ^{14}C atoms per gram of carbon in modern wood charcoal.

$A = kN \quad A = .255\ Bq \quad k = \frac{\ln(2)}{t_{1/2}} = \frac{\ln(2)}{5730\,yr} = 1.21\times10^{-4}\ yr^{-1}$

$N = \frac{A}{k} \quad k = \frac{1.21\times10^{-4}}{yr}\cdot\frac{1\ yr}{365\ days}\cdot\frac{1\ day}{24\ hrs}\cdot\frac{1\ hr}{3600\ s}$

$k = 3.84\times10^{-12}\ s^{-1}$

$N = \frac{.255\ dis.\cdot s}{3.84\times10^{-12}\ s}$

$$\boxed{N = 6.65\times10^{10}\ \frac{\text{atoms of } C^{14}}{\text{gram wood}}}$$

(b) Calculate the fraction of carbon atoms in the biosphere that are ^{14}C. *Assume charcoal is representative of the isotopic composition of carbon throughout the biosphere.*

$\text{Faction} = \frac{\#\text{ of } ^{14}C \text{ atoms in wood charcoal}}{\text{total }\#\text{ of carbon atoms in wood charcoal}}$

$6.65\times10^{10}\ \text{atoms } ^{14}C \cdot \frac{\text{mole}}{6.022\times10^{23}\ \text{atoms}}\cdot\frac{14.00\ g}{1\ \text{mole}} = 1.55\times10^{-12}\ \text{grams}$

$1g - 1.55\times10^{-12}\ g \simeq 1\ g$

$1\ \text{gram charcoal}\cdot\frac{1\ \text{mole}}{12.00\ g\ \text{Charcoal}}\cdot\frac{6.022\times10^{23}\ \text{atoms}}{1\ \text{mole}} = 5.02\times10^{22}\ \text{atoms C total}$

$\text{Fraction} = \frac{6.65\times10^{10}\ \text{atoms } ^{14}C}{5.02\times10^{22}\ \text{atoms C}} = \boxed{1.32\times10^{-12}\ \frac{\text{atoms } ^{14}C}{\text{atoms Carbon}}}$

3. (10 + 10 points) The nuclide ^{131}I undergoes beta decay with a half-life of 8.041 days. Large quantities of this nuclide were released into the environment in the Chernobyl accident. A victim of radiation poisoning has absorbed 5.0×10^{-6} g of ^{131}I.

(a) Compute the activity, in becquerels (Bq), of ^{131}I in the victim, taking the atomic mass of the nuclide to equal 131 g mol^{-1}.

$A = kN$

$k = \frac{\ln(2)}{t_{1/2}} = \frac{\ln(2)}{8.041 \text{ days}} = 8.62 \times 10^{-2} \text{ days}^{-1}$

$N = 5.0 \times 10^{-6} \text{ g} \cdot \frac{\text{mole } ^{131}I}{131 \text{ g } ^{131}I} \cdot \frac{6.022 \times 10^{23} \text{ atoms}}{1 \text{ mole}} = 2.30 \times 10^{16}$ atoms

$k = \frac{8.62 \times 10^{-2}}{\text{day}} \cdot \frac{1 \text{ day}}{24 \text{ hrs}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} = 9.98 \times 10^{-7} \text{ s}^{-1}$

$A = (9.98 \times 10^{-7} \text{ s}^{-1})(2.30 \times 10^{16} \text{ atoms})$

$\boxed{A = 2.29 \times 10^{10} \text{ Bq}}$

(b) Compute the radiation absorbed dose, in *milligrays* (mGy), caused by this nuclide during the first second after its ingestion. Assume beta particles emitted by ^{131}I have an average kinetic energy of 0.40 MeV, that all of this energy is deposited within the victim's body, *the change in decay rate in the first second is negligible,* and that the victim weights 60 kg.

$1 \text{ Gray} = 1000 \text{ mGy} = 100 \text{ rad} = \frac{1 \text{ J}}{\text{kg}}$

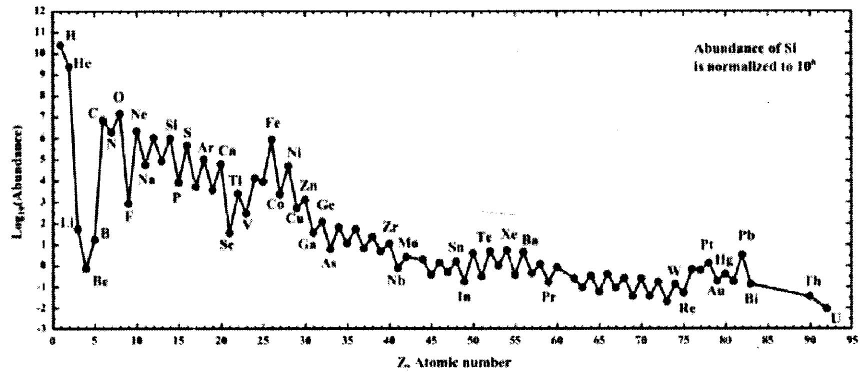
$A \downarrow$

$2.29 \times 10^{10} \frac{\text{dis}}{\text{s}} \cdot 1 \text{ s} \cdot \frac{.40 \text{ MeV}}{\text{dis.}} \cdot \frac{1.602 \times 10^{-13} \text{ J}}{1 \text{ MeV}} \cdot \frac{1}{60 \text{ kg}} =$

$2.45 \times 10^{-5} \frac{\text{J}}{\text{kg}}$

$2.45 \times 10^{-5} \frac{\text{J}}{\text{kg}} \cdot \frac{1 \text{ Gy}}{1 \text{ J/kg}} \cdot \frac{1000 \text{ MGy}}{1 \text{ Gy}} = \boxed{2.45 \times 10^{-2} \text{ MGy}}$

4. (5 + 5 + 10 points) Consider the cosmic abundance of the elements.



(a) Briefly explain why hydrogen (H) and helium (He) are so abundant.

During Big Bang, Hydrogen and Helium were the two main elements created and are the building blocks from which other elements are synthesized.

(b) Briefly explain the even-odd alteration in cosmic abundance with atomic number.

$^{4}_{2}He$ is the building block for the synthesis of larger nuclei and Helium has an even number so it is more likely that the atomic number of larger nuclei will be a multiple of Heliums atomic number (2).

(c) As main sequence stars age and accumulate helium, they begin to contract and the temperature increases to ~10^8 K. Write the relevant nuclear reaction(s) that are believed to occur to form stable ^{12}C.

$$2\ ^{4}_{2}He \longleftrightarrow ^{8}_{4}Be$$

$$^{8}_{4}Be + ^{4}_{2}He \rightarrow ^{12}_{6}C$$

5. (10 points) An I_2 (s)|I^- (1.00 M) half-cell is connected to an H_3O^+|H_2 (1 atm) half-cell in which the concentration of the hydronium ion is unknown. The measured cell potential is 0.841 V, and the I_2|I^- half-cell is the cathode. What is the pH in the H_3O^+|H_2 half-cell at 25 °C?

$E_{measure} = .841\ V$

(cathode) $I_{2(s)} + 2e^- \rightarrow 2I^-_{(aq)}$ $E^\circ = .535\ V$

(anode) $H_{2(g)} \rightarrow 2H^+_{(aq)} + 2e^-$ $E^\circ = 0.00\ V$

$I_{2(s)} + H_{2(g)} \longrightarrow 2H^+_{(aq)} + 2I^-_{(aq)}$

$$E^\circ_{cell} - \frac{.0592V}{n} \log Q = E_{measured}$$

$E^\circ_{cell} = E_{cath} - E_{anode} = .535V - 0.00V = .535V$ $n = 2\ mole\ e^-$

$$.535\ V - \frac{.0592V}{2\ mole\ e^-} \log\left(\frac{[H^+]^2[I^-]^2}{(P_{H_2})}\right) = .841\ V$$

$$(.841\ V - .535V) = -\frac{.0592V}{2\ mol\ e^-} \log\left(\frac{[H^+]^2[1.00M]^2}{1.00\ atm}\right)$$

$$\frac{(.841V - .535V)}{.0592\ V} = -\log[H^+] \leftarrow -\log[H^+] = pH$$

$$\frac{.841\ V - .535\ V}{.0592\ V} = pH$$

$$\boxed{pH = 5.17}$$

6. (10 points) A Ni|Ni^{2+}||Ag^{+}|Ag galvanic cell is constructed in which the standard cell potential is 1.03 V. Calculate the free energy change at 25°C when 1.00 g of silver plates out, if all concentrations remain at their standard value of 1 M throughout the process. What is the maximum electrical work *done by the cell* on its surroundings during this experiment?

$E^{0}_{cell} = 1.03\ V \quad \Delta G^{0} = -nF\Delta E^{0}_{cell} = w_{max}$

(cath) $2Ag^{+}_{(aq)} + 2e^{-} \rightarrow 2Ag_{(s)}$

(anod) $Ni_{(s)} \rightarrow Ni^{+2}_{(aq)} + 2e^{-}$

$1.00\ g_{Ag} \cdot \frac{1\ mole\ Ag}{107.47\ g\ Ag} \cdot \frac{1\ mole\ e^{-}}{1\ mole\ Ag} = 9.27 \times 10^{-3}$ moles e^{-} required to plate 1.00g $Ag_{(s)}$

$\Delta G = -(9.27 \times 10^{-3}\ mole\ e^{-})\left(96{,}485\ \frac{C}{mol\ e^{-}}\right)(1.03\ V)$

$\boxed{\Delta G = -9.21 \times 10^{2}\ J}$

$\Delta G = -w_{on\ surrounding}$

$\boxed{W_{(on\ surroundings)} = 9.21 \times 10^{2}\ J}$

NO TEST MATERIAL ON THIS PAGE