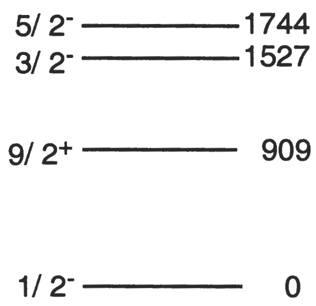
Department of Nuclear Engineering
University of California
Berkeley, California

NE 101
Fall 2003
Second Midterm
Open Book

1. (16 points) The ground and first three excited states of ${}^{89}_{39}Y$ are shown below, the spins and parities to the left and the level energies in keV to the right.



a. Use the single particle level diagram to determine the proton particle configurations that give rise to these states. The particle configuration is given by writing down each of the level names, such as $1s_{1/2}$, $3d_{5/2}$, etc., and indicating the number of particles in each state, such as $1s^2_{1/2}$, $3d^3_{1/2}$, etc. For this problem you should include all of the levels for the protons between the closed shells of Z = 28 and Z = 50 to describe the configurations of the 4 levels shown above. To get the configuration for each of the excited states, start from that for the ground state and move a single particle so that you get the correct spin and parity with the minimum energy increase.

b. Write down or derive an expression for the energy difference between the ground and <u>second</u> excited states of ${}^{89}_{39}Y$ in terms of the energies of the single particle states involved and the pairing energies associated with them. Use the terminology ε_i and δ_i to represent the level energy and the pairing energy in the ith single particle state. For this part you only need be concerned directly with the two single particle states involved.

2. (15 points) The excitation energy of the first excited state of a hypothetical even,even nucleus is 0.050 MeV.

a. Is the nucleus likely to be spherical or well-deformed? For credit you must give a brief explanation for your choice.

b. Assuming that the nucleus is well-deformed, estimate the energies of the 2nd and 3rd excited states of this nucleus.

c. Discuss whether the energies calculated in part b are expected to be over or underestimates of the actual energies found in a real nucleus.

3. (12 points) A hypothetical even,even nucleus can be described as doubly magic plus two identical particles that reside in a $i_{13/2}$ orbital.

a. How many identical particles can occupy the $i_{13/2}$ orbital?

b. What are the spins and parities of all of the levels expected from the coupling of two identical particles in the $i_{13/2}$ orbital?

c. Of the set of levels given in part b, give (and identify which is which) the spins and parities of the lowest- and highest-energy members of the set.

4. (15 points)

a. Draw a potential diagram for a neutron of orbital angular momentum $l_n > 0$ interacting with a nucleus represented by a static potential well of the form

$$V(r) = \begin{cases} -V_o, \ (r < R_n) \\ 0, \ r \geq R_n \end{cases}$$

Add a horizontal line representing a small positive total neutron energy E_n such that $0 < E_n < V_{tot}(R_n)$, where $V_{tot}(R_n)$ is the total potential at $r = R_n$. You can treat the neutron as a point particle. Finally label the radial coordinate where $E_n = V_{tot}(b)$ with the symbol b.

b. Let $< T >$ be the average neutron kinetic energy while contained within the well with $r \leq b$. What is the relation between $< T >$, $V_{tot}(R_n)$ for $r \leq R_n$, and E_n? Write an accurate mathematical expression for our answer.

c. Now use the fundamental concepts developed in the simple theory of α decay to write down an expression from which the decay constant for neutron emission can be estimated. Remember that the transmission coefficient depends on both the velocity of the neutron after being emitted and when it is far from the nucleus as well as the velocity of the neutron while it is confined to the well. You must use the complete expression for the centrifugal potential in your answer.

5. (12 points)

a. A hypothetical odd,odd nucleus has the odd neutron in a $d_{5/2}$ orbital and the odd proton in a $f_{3/2}$ orbital. What are the spins and parities of the possible states from this configuration?

b. An odd-A nucleus decays by an α transition from a $5/2^-$ level to a $3/2^+$ level in the daughter nucleus. What are the allowed angular momenta for the emitted α particles?

c. Assume that you have a nucleus of large Z in a highly-excited state that can decay by either neutron or proton emission of exactly the same energies. Experimentally, it is observed that the nucleus will almost always decay by neutron emission. Give an explanation for this fact. Use a potential diagram if you can in your answer.

6. (15 points) The ground state of the deuteron, ${}^{2}_{1}H$ is known to have a wave function of the form

$$\psi = |0.9798|\psi(l = 0) + |0.2|\psi(l = 2),$$

where the signs of the coefficients have been neglected and l refers to the inferred relative orbital angular momentum of the neutron and proton based on experimental measurements.

a. Using this notation, what would be the wave function in the extreme single particle model that we have studied in this course?

b. Is the wave function normalized?

c. Is the fact that the wave function has a component of l = 2 consistent with a definite parity of the ground state of the deuteron?

d. What is implied by the single particle states that are occupied by the neutron and proton by this wave function? Look at the single particle level diagram to help in getting your answer.

7. (15 points) Binary nuclear fission takes place according to

$$M(Z, A) = M(Z_H, A_H) + M(Z_L, A_L).$$

If the energy release is Q_f, derive an expression for the ratio of the kinetic energy of the fission products, neglecting neutron or γ-ray emission.