Math 55

PROFESSOR KENNETH A. RIBET

First Midterm Examination
February 21, 2019
12:40–2:00PM, 2050 VLSB

NAME:	SID:

Circle your GSI's name: **Katrina Biele** **Ben Castle** **Rahul Dalal**
Christopher Miller **Max Wimberley** **Dylan Yott** **Dongxiao Yu**

At the conclusion of the exam, hand your paper in to your GSI.

Please put away all books, calculators, cell phones and other devices. You may consult a single two-sided sheet of notes. Please write carefully and clearly, *USING SENTENCES* (not just symbols). Remember that the paper you hand in will be your only representative when your work is graded. Please write your name clearly on each page of your exam. Your paper will be scanned and will be processed using Gradescope. It is essential that you hand in all pages that you have received (including this cover sheet) and that the order of the pages be preserved. You do *not* need to hand in the sheet of notes that you brought with you to the exam. You do *not* need to simplify arithmetic expressions.

Your booklet should contain this cover page and six pages with problems on them; there are seven pages in all. The backs of all pages may be used for scratchwork or for continuations of your answers. If a solution spans more than one page, please indicate that clearly so that the solution will be read in its entirety.

Problem	1	2	3	4	5	6	Total
Points	6	6	5	6	5	6	34

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1a. If S is a subset of a countable set, is S necessarily countable? Explain your answer carefully, outlining a proof or giving a counterexample.

b. Suppose that $f : T \to \{1, 2, 3, \ldots\}$ is an onto function. Is the set T necessarily countable?

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2. Using mathematical and logical operators, predicates, and quantifiers (where the domain consists of all integers) express: "The difference of two positive integers is not necessarily positive."

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3. Prove or disprove: if A and B are sets, then $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$.

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4. Use the Euclidean algorithm to find the gcd of 39 and 57 and to write the gcd as a linear combination of 39 and 57.

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5. Find the smallest non-negative integer satisfying the three congruences

$$x \equiv \begin{cases} -3 \mod 19 \\ -3 \mod 20 \\ -3 \mod 21. \end{cases}$$

(Explain carefully how you got your result.)

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6. Use Bézout's theorem to prove that if a is relatively prime both to b and to c, then a is relatively prime to bc. In symbols:

$$\gcd(a,b) = \gcd(a,c) = 1 \quad \stackrel{?}{\longrightarrow} \quad \gcd(a,bc) = 1.$$

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