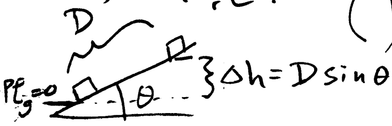
Mike DeWeese MT2

4. a) $PE_{sp} = \frac{1}{2}k\Delta x^2 = \boxed{\frac{1}{2}k(L-\ell)^2}$

b.) C.O.E.: $(\cancelto{0}{KE_f} + \cancelto{0}{PE_{sp,f}} + PE_{g,f})_f - (\cancelto{0}{KE_i} + PE_{sp,i} + \cancelto{0}{PE_{g,i}})_i = \cancelto{0}{W_{ext}}$ (no friction)

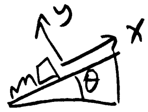


$$mgD\sin\theta - \frac{1}{2}k(L-\ell)^2 = 0$$

$$D = \boxed{\frac{k(L-\ell)^2}{2mg\sin\theta}}$$

c.) $W_g = \vec{F}_g \cdot \Delta\vec{r}_{block} = \boxed{-mg\sin\theta(L-\ell)}$

d.) $\vec{J} = \Delta\vec{P} = \vec{P}_f - \cancelto{0}{\vec{P}_i}$ — ①



C.O.E. $\left(\frac{1}{2}mv_f^2 + mg(L-\ell)\sin\theta\right)_f - \left(\frac{1}{2}k(L-\ell)^2\right)_i = \cancelto{0}{W_{ext}}$

$$v_f = \sqrt{-2g(L-\ell)\sin\theta + \frac{k}{m}(L-\ell)^2} \quad \text{— ②}$$

② in ① $\Rightarrow$ $\boxed{\vec{J} = m\sqrt{-2g(L-\ell)\sin\theta + \frac{k}{m}(L-\ell)^2}\ \hat{i}}$

$\vec{J}$ points up the slope

e.) C.O.E. $(\cancelto{0}{KE_f} + \cancelto{0}{PE_{sp}} + mgD'\sin\theta)_f - (\cancelto{0}{KE_i} + \frac{1}{2}k(L-\ell)^2 + \cancelto{0}{PE_{g,i}}) = -D'\mu_k F_N$ ③

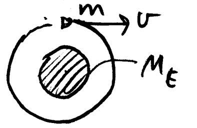
N2L y (block): $F_N - mg\cos\theta = m\cancelto{0}{a_y} \Rightarrow F_N = mg\cos\theta$ — ④

④ in ③ $\Rightarrow$ $mgD'\sin\theta - \frac{1}{2}k(L-\ell)^2 = -D'\mu_k mg\cos\theta$

$$D'mg(\sin\theta + \mu_k\cos\theta) = \frac{1}{2}k(L-\ell)^2$$

$$\boxed{D' = \frac{k(L-\ell)^2}{2mg(\sin\theta + \mu_k\cos\theta)}}$$

2. a.) uniform circular motion: $a_y = -\frac{v^2}{r} = -\frac{v^2}{2R}$



$\uparrow y$ N2L projectile y: $F_{net,y} = m a_y$

$$-G\frac{\cancel{m}M_E}{(2R)^2} = \cancel{m}\left(-\frac{v^2}{2R}\right)$$

$$\boxed{v = \sqrt{\frac{GM_E}{2R}}}$$

b.) $T = \frac{2\pi(2R)}{v} = \boxed{4\pi R\sqrt{\frac{2R}{GM_E}}}$

c.) C.O.E.: $\left(-G\frac{M_E m}{r_{max}} + \cancel{KE_f}^{\,0}\right)_f - \left(-\frac{GM_E m}{2R} + \frac{1}{2}m\frac{GM_E}{2R}\right)_i = 0$

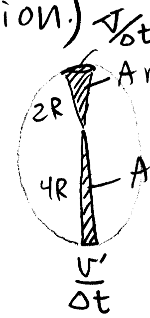
$$\frac{1}{r_{max}} = \frac{1}{4R}$$

$$r_{max} = \boxed{4R}$$

d.) K2L: equal area is swept out by a line connecting the sun (or Earth in this case) and a given planet (or projectile) at any point in its orbit.

(OR: Conservation of angular momentum: Central forces such as gravity can't exert a torque on the orbiting object, so $\vec{L} = \text{const.}$ throughout the motion.)

$\uparrow y$



$\text{Area} \approx \frac{1}{2}2R\frac{V}{\Delta t} = \frac{RV}{\Delta t}$

$\text{Area} \approx \frac{1}{2}4R\frac{v'}{\Delta t} = \frac{2Rv'}{\Delta t}$

$\Big\}\ v' = \frac{V}{2}$

OR: C.o.$\vec{L}$

$V2R = v'4R \Rightarrow v' = \frac{V}{2}$

C.O.E. $-\frac{G\cancel{m}M_E}{2R} + \frac{1}{2}\cancel{m}V^2 = -\frac{G\cancel{m}M_E}{4R} + \frac{1}{2}\cancel{m}\frac{V^2}{4}$

$$\frac{1}{2}\frac{3}{4}V^2 = \frac{GM_E}{4R}$$

$$\boxed{V = \sqrt{\frac{2}{3}\frac{GM_E}{R}}}$$

e.)

K3L: $\frac{a_1^3}{T_1^2} = \frac{a_2^3}{T_2^2}$

$a \equiv$ semi-major axis $= 3R$

$\frac{(3R)^3}{T^2} = \frac{(6R)^3}{(1\,\text{day})^2}$ $T = \left(\frac{1}{2}\right)^{3/2}$ days $= \boxed{\frac{1}{2\sqrt{2}}\text{ days}}$

3. a.) c.o.$\vec{P}$. $\quad M(v_{s,x}-v)+2Mv_{s,x}=0$

$$3v_{s,x}=v$$
$$v_{s,x}=\boxed{\tfrac{1}{3}v}$$

b.) $\quad M(v'_{s,x}-v)+Mv'_{s,x}=2M\tfrac{1}{3}v$

$$2v'_{s,x}=\left(\tfrac{2}{3}+1\right)v$$
$$v'_{s,x}=\boxed{\tfrac{5}{6}v}$$

c.) $\quad M(v''_{s,x}+v)+Mv''_{s,x}=2M\tfrac{1}{3}v$

boy jumps rightward

$$2v''_{s,x}=\left(\tfrac{2}{3}-1\right)v$$
$$v''_{s,x}=\boxed{-\tfrac{1}{6}v}<0 \Rightarrow \text{sled moves to left in diagram}$$

d.) c.o.$\vec{P}$. $\quad 2Mv_{sxf}=MV+\cancel{M0}^{\,0}$

snow is initially at rest

$$v_{sxf}=\boxed{\tfrac{1}{2}V}$$

e.) variable mass problem: $\quad \overset{\vec{u}}{(dm)}+\overset{\vec{v}}{(M)}\rightarrow \overset{\vec{v}+d\vec{v}}{(M+dM)}$

$$d\vec{P}=(M+dM)(\vec{v}+d\vec{v})-dM\vec{u}-M\vec{v}$$
$$=\cancel{M\vec{v}}+dM\vec{v}+Md\vec{v}+\underbrace{dMd\vec{v}}_{\approx 0}-dM\vec{u}-\cancel{M\vec{v}}$$
$$=Md\vec{v}-(\vec{u}-\vec{v})dM$$

$$\vec{F}_{ext}=\frac{d\vec{P}}{dt}=M\frac{d\vec{v}}{dt}-(\vec{u}-\vec{v})\frac{dM}{dt}$$

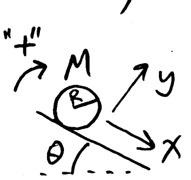
$$F_{ext_x}=0\Rightarrow 0=M\frac{dv_x}{dt}-\underbrace{(u_x-v_x)}_{\text{relative velocity of snowballs w/r sled}=\text{const}=v_{sb}}\frac{dM}{dt}$$

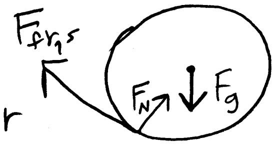
$$\int_{v_{0,x}}^{v_{f,x}}dv=\int_{M_0}^{M_f}\frac{dM}{M}v_{sb}$$

$$v_f-0=-v_{sb}\ln\left(\frac{M_f}{M_0}\right)=-v_{sb}\ln\left(\frac{2M}{3M}\right)=\boxed{v_{sb}\ln(3/2)}$$

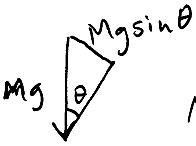
snowballs thrown leftward

4. a.) FBD cylinder





b.) N2L x: $F_{net_x} = M a_{cm_x}$



$$\cancel{F_{N_x}}^{0} + Mg\sin\theta - F_{fr,s} = M a_{cm_x} \quad (1)$$

N2L α: $\tau_{net_{cm}} = I_{cm}\alpha$

$$\cancel{\tau_{N_{cm}}}^{0} + \cancel{\tau_{g_{cm}}}^{0} + F_{fr,s}R = \frac{1}{2}MR^2\alpha \quad (2)$$

rolling without slipping: $a_{cm_x} = +R\alpha$ (3)

(1) & (3) = $Mg\sin\theta - F_{fr,s} = MR\alpha$ (4)

(2) & (4) ⇒ $(\cancel{M}g\sin\theta - \cancel{M}R\alpha)R = \frac{1}{2}\cancel{M}R^2\alpha$

$$\alpha \frac{3}{2}R = g\sin\theta$$

$$\boxed{\alpha = \frac{2}{3}\frac{g\sin\theta}{R}} > 0 \Rightarrow \alpha \text{ is } \underline{\text{clockwise}} \quad (5)$$

c.) static friction: $F_{fr,s} \le \mu_s F_N$ (6)

N2L y: $F_N - Mg\cos\theta = M\cancel{a_{y_{cm}}}^{0} \Rightarrow F_N = Mg\cos\theta$ (7)

(2) & (5) ⇒ $F_{fr,s} = \frac{1}{2}M\cancel{R}\frac{2}{3}g\frac{\sin\theta}{\cancel{R}} = \frac{1}{3}Mg\sin\theta$ (8)

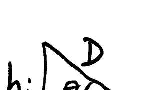
(6) & (7) & (8) ⇒ $\frac{1}{3}\cancel{Mg}\sin\theta \le \mu_s \cancel{Mg}\cos\theta$

$$\boxed{\mu_s \ge \frac{1}{3}\frac{\sin\theta}{\cos\theta} = \frac{1}{3}\tan\theta}$$

d.) C.O.E.: $(KE_{f_{tot}} + Mgh_f)_f - (\cancel{KE_{i_{tot}}}^{0} + Mgh_i)_i = \cancel{W_{ext}}^{0}$

0 starts from rest

$F_{fr,s}$ does no work since point of contact is not moving



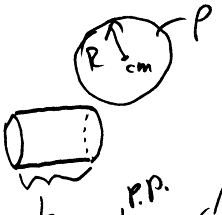
$$\frac{1}{2}Mv_{cm_f}^2 + \frac{1}{2}I_{cm}\omega_f^2 - MgD\sin\theta = 0 \quad (9)$$

rolling without slipping ⇒ $v_{cm_f} = R\omega_f$ (10)

(9) & (10) ⇒ $\frac{1}{2}\cancel{M}v_{cm_f}^2 + \frac{1}{2}\frac{1}{2}\cancel{M}\cancel{R^2}\frac{v_{cm_f}^2}{\cancel{R^2}} = \cancel{M}gD\sin\theta$

$$\frac{3}{4}v_{cm_f}^2 = gD\sin\theta \Rightarrow \boxed{v_{cm_f} = \sqrt{\frac{4}{3}gD\sin\theta}}$$

4.e.) I_{cm} of solid cylinder w/ uniform density ρ, radius R



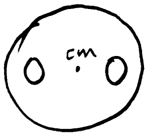
$$I_{cm,\ \text{solid cylinder}} = \frac{1}{2} M R^2 = \frac{1}{2}(\rho \pi R^2 L) R^2 = \frac{1}{2}\pi \rho L R^4$$

parallel axis theorem

$$I_{pp,\ \text{solid cylinder}} \overset{\Downarrow}{=} I_{cm} + M'd^2 = \frac{1}{2}(\rho \pi r^2 L) r^2 + (\rho \pi r^2 L) d^2$$

$$= \rho \pi r^2 L \left(\frac{r^2}{2} + d^2\right)$$

so for cylinder with 2 holes drilled out:



$$I_{cm} = I_{cm,\ \text{solid cylinder}} - 2 I_{pp,\ \text{small solid cylinder}}$$

$$= \frac{1}{2}\pi \rho L R^4 - 2 \rho \pi r^2 L \left(\frac{r^2}{2} + d^2\right)$$

$$= \boxed{\rho L \pi \left(\frac{R^4}{2} - r^4 - 2 r^2 d^2\right)}$$