Fall 2015 Final Solutions

Physics 8A

Problem 1 Heat will flow from the warm end of the rod into the ice bath. We are told the warm end of the rod is held at a constant temperature. Since the ice bath is a combination of ice and water, it is at an initial temperature of 0 C°, and its temperature while melting will also stay constant. While going through a phase change water's temperature stays constant at 0 C°, this is a property of phase changes. Since both ends of the rod will be at constant temperatures, the heat flow during the melting process will also be constant.
The main equation we will use is the heat flow equation.

$$H = \frac{-kA}{\Delta x}\Delta T$$

$$A = 1 * 10^{-4}\ m^2$$

$$\Delta x = L = 1\ m$$

$$k = 0.2\ \frac{W}{mK}$$

$$\Delta T = (T_{ice} - T_{warm}) = 0 - 50 = -50K$$

Plugging all of this in to the heat flow equation,

$$H = \frac{-0.2 * 1 * 10^{-4}}{1}(-50)$$

$$H = 10^{-3}\ W$$

Now that we know how fast heat is being transferred to the ice, I can figure out how much heat is needed to melt the ice. Once I know that, I can figure out how long it takes to transfer that much heat with the 10^{-3} heat flow.

$$Q = mL$$

$$Q = 1\ kg\ * 300 * 10^3\ \frac{J}{kg}$$

$$Q = 300 * 10^3\ J$$

The time is takes to transfer this much heat is given by,

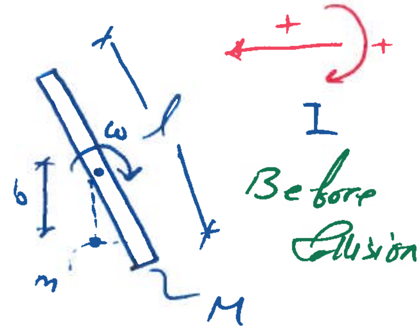
$$t = \frac{Q}{H}$$

$$t = \frac{300 * 10^3}{10^{-3}}$$

$$t = 300 * 10^6\ s$$

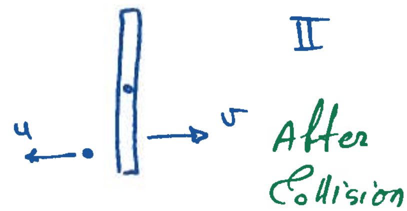
[redacted] I: Angular momentum: $\overset{\text{Rod}}{I\omega} + \overset{\text{mass}}{0}$

Linear momentum: $0 + 0$



II: Angular momentum: $\overset{\text{Rod}}{0\cancel{(Mv)}} + \overset{\text{mass}}{b(mu)}$

Linear momentum : $-Mv + mu$



Conservation of angular momentum:

$$\underset{\text{I}}{I\omega} = \underset{\text{II}}{bmu} \Rightarrow \frac{1}{12}Ml^2\omega = bmu$$

$$\boxed{u = \frac{1}{12}\frac{M}{m}\frac{l^2}{b}\omega}$$

Conservation of linear momentum:

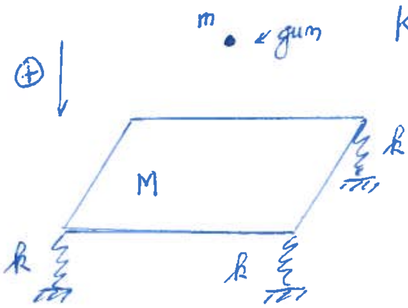
$$\underset{\text{I}}{0} = \underset{\text{II}}{-Mv + mu} \Rightarrow v = \frac{m}{M}u$$

$$= \frac{m}{M}\left(\frac{1}{12}\frac{M}{m}\frac{l^2}{b}\omega\right)$$

$$\Longrightarrow \boxed{v = \frac{1}{12}\frac{l^2}{b}\omega}$$

We are not using Conservation of Kinetic Energy, because we don't know if the collision is totally elastic.

3.) Total stiffness :



$$K = \sum_{1}^{4} k_i = 4k$$

a.) Conservation of energy gives :

$$E_1 = E_2$$

index 1 : gum @ height h

2 : gum @ plate before the collision

$$mgh_1 + \cancel{\frac{1}{2} m v_1^2}^{\,0} = \cancel{mgh_2}^{\,0} + \frac{1}{2} m v_2^2$$

$$\Rightarrow \quad v_2 = \sqrt{2gh} \qquad \text{where } h_1 = h$$

Conservation of momentum gives :

$$\sum \vec{P_2} = \sum \vec{P_3}$$

index 2 : gum & plate before collision

3 : gum & plate after collision

$$m\vec{v_2} + \cancel{M\vec{v}_{2\,plate}}^{\,\vec{0}} = (m+M)\vec{v_3}$$

or $$m\vec{v_2} = (m+M)\vec{v_3}$$

On possitive direction ⊕ :

$$mv_2 = (m+M)v_3$$

$$\Rightarrow \quad v_3 = \frac{mv_2}{m+M} = \frac{m\sqrt{2gh}}{m+M}$$

b.) Angular frequency for the system of mass & spring :

$$\omega = \sqrt{\frac{\text{stiffness}}{\text{mass}}} \qquad \left(\sqrt{\frac{K}{m}} \text{ in lecture notes} \right)$$

we have : stiffness $= 4k$

mass $= m + M$

$$\Rightarrow \quad \omega = \sqrt{\frac{4k}{m+M}}$$

Problem 4

a) For simplicity, define the direction into the plane as positive direction.

Acceleration of the center of mass

$$a = \frac{F}{M}$$

torque acting on the sphere

$$\tau = FR = I\alpha$$

and

$$I = \frac{2}{5}MR^2$$

thus,

$$\alpha = \frac{5F}{2MR}$$

b) distance that the center of mass of the sphere moves

$$D = \frac{1}{2}at^2 = \frac{Ft^2}{2M}$$

angle that the sphere rotates about its center

$$\theta = \frac{1}{2}\alpha t^2 = \frac{5Ft^2}{4MR}$$

then, the total length of string unwound when traveling the distance D

$$L = \theta R = \frac{5Ft^2}{4M}$$

thus,

$$\frac{D}{L} = \frac{\frac{Ft^2}{2M}}{\frac{5Ft^2}{4M}} = \frac{2}{5}$$

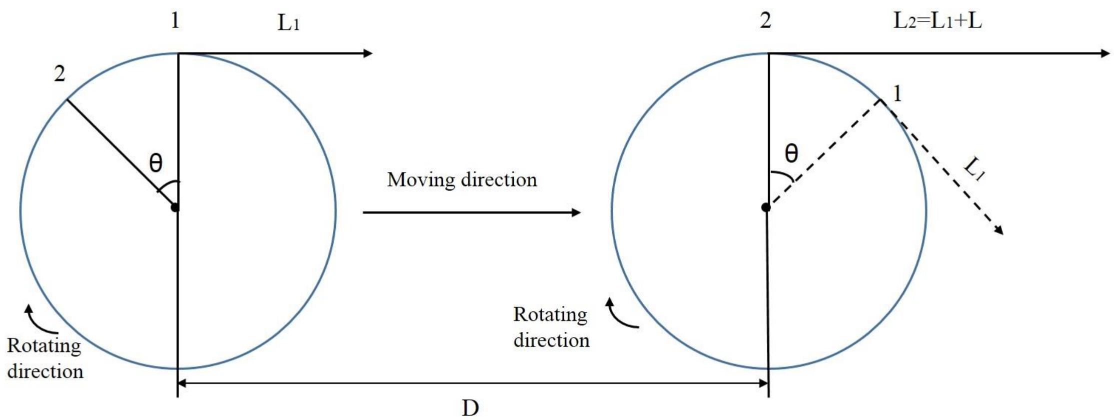


Figure 1. Top view of the motion

5. a) Because we are given both the distance the block travels vertically and horizontally once it leaves the table/ramp, the only way to solve this is with kinematics (not with frictional forces). First find the time it would take the block to fall $H = 1.0m$ and use that to find the x-velocity as it leaves the table.

Y: $y_f - \cancel{y_i}^{0} = \cancel{V_y}^{0} t + \frac{1}{2} a_y t^2$

$-1.0m = -\frac{1}{2} g t^2$

$t^2 = \frac{2m}{g}$

$t = \sqrt{\frac{2m}{g}}$

X: $x_f - \cancel{x_i}^{0} = V_x t + \frac{1}{2} \cancel{a_x}^{0} t^2$

$1m = V_0 t$

$V_0 = \frac{1m}{t} = \frac{1m}{\sqrt{2m/g}}$

$V_0 = \sqrt{\frac{g m^2}{2m}} = \sqrt{\frac{g m}{2}}$

$\boxed{V_0 = \sqrt{4.9 \frac{m^2}{s^2}}}$

b) This is a conservation of energy problem

$E_{total\,initial} + W_{ork} = E_{Total-final}$

$PE_i + W_f = KE_f$

$mgh + W_f = \frac{1}{2} m V_0^2$

$W_f = \frac{1}{2} m V_0^2 - mgh$

$= m\left[\frac{1}{2} V_0^2 - gh\right]$

$= (1kg)\left[\frac{1}{2}\left(\frac{g}{2} m\right) - g\left(\frac{1}{2} m\right)\right]$

$= 1kg\left[\frac{1}{4} g - \frac{1}{2} g\right] m = -\frac{1}{4} g (kg\,m)$

$\boxed{= -\frac{1}{4} 9.8 J = -2.45 J}$

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(a)

$$P_2V_2^\gamma = P_3V_3^\gamma$$
$$P_3 = \frac{P_2V_2^\gamma}{V_3^\gamma} = 2\times 10^5 \times (\frac{10^{-4}}{10})^{1.4} = 2\times 10^{-2}Pa$$
$$P_1V_1^\gamma = P_4V_4^\gamma$$
$$P_4 = \frac{P_1V_1^\gamma}{V_4^\gamma} = 1\times 10^5 \times (\frac{10^{-4}}{10})^{1.4} = 1\times 10^{-2}Pa$$

The total work done in the cycle is,

$$\begin{aligned} W &= W_{2-3} + W_{4-1} \\ &= \frac{P_3V_3 - P_2V_2}{1-\gamma} + \frac{P_1V_1 - P_4V_4}{1-\gamma} \\ &= 24.75\ J \end{aligned}$$

(b)

$$Q_h = Q_{1-2} = nC_V\Delta T$$

Since $PV = nRT$, we have

$$\begin{aligned} Q_h &= \frac{C_V}{R}\Delta PV \\ &= \frac{5}{2}(2\times 10^5 - 1\times 10^5)\times 10^{-4} \\ &= 25\ J \end{aligned}$$

(c)

$$e = \frac{W}{Q_h} = \frac{24.75\ J}{25\ J} = 99\%$$