Q 1 (a) [2]

$$\downarrow mg\left(\underline{E_x}\sin\alpha - \underline{E_y}\cos\alpha\right)$$

① point: only force is weight $m\underline{g}$

① point: direction of weight in terms of $\underline{E_x}$, $\underline{E_y}$ & α

(b) [5] $\underline{r} = x\,\underline{E_x} + y\,\underline{E_y}$ $\qquad \ddot{\underline{r}} = \underline{a} \qquad \underline{F} = m\underline{a}$

$$\rightarrow \; mg(\underline{E_x}\sin\alpha - \underline{E_y}\cos\alpha) = m\ddot{x}\,\underline{E_x} + m\ddot{y}\,\underline{E_y}$$ ②

Impact at $t = T$: $y(T) = 0$ $\Rightarrow$

$$\ddot{y} = -g\cos\alpha$$
$$\dot{y}(t) = \dot{y}(0) + \int_0^t -g\cos\alpha\, d\tau$$
$$= v_{0y} - g\,t\cos\alpha$$
$$y(t) = y(0) + \int_0^t \dot{y}(\tau)\, d\tau$$
$$= 0 + v_{0y}t - \tfrac{1}{2}gt^2\cos\alpha$$

② correct & formal integration

$$y(T) = 0 \Rightarrow v_{0y}T - \tfrac{1}{2}gT^2\cos\alpha = 0, \quad T > 0$$

① point for argument

$$\rightarrow \; T = \frac{2v_{0y}}{g\cos\alpha}$$

(c) [3] Coordinates $\quad \underline{r}(t) = x(T)\,\underline{E_x} + 0\,\underline{E_y}$

$$\dot{x}(t) = \cancel{\dot{x}(0)}^{\,0} + g\sin\alpha\, t \qquad x(t) = x(0) + \tfrac{1}{2}gt^2\sin\alpha$$

② for integrating to $x(t)$

$$x(T) = \frac{\cancel{g}}{\cancel{2}}\,\frac{2^{\cancel{2}}\,v_{0y}^2}{g^{\cancel{2}}\cos^2\alpha}\sin\alpha = \frac{2v_{0y}^2}{g\cos^2\alpha}\cdot\sin\alpha$$

① for answer

Q2 (a) [5]

$$\underline{r} = x\,\underline{E}_x - x^2\,\underline{E}_y$$

$$\underline{v} = \frac{d}{dt}\underline{r} = \frac{dx}{dt}\frac{d}{dx}\underline{r} = \dot{x}\left(\underline{E}_x - 2x\,\underline{E}_y\right)$$ 2

$$v = \|\underline{v}\| = |\dot{x}|\sqrt{1+4x^2}$$ 2 Must have abs. value for $\dot{x}$, or $\sqrt{\dot{x}^2}$

But, we know $\dot{x} > 0$ (left-to-right) 1 ↳ Must be explicit about why we drop abs. val.

$$\Rightarrow \quad \dot{x} = \frac{v}{\sqrt{1+4x^2}}$$

Max horizontal speed: $\dfrac{d\dot{x}}{dx} = -\frac{1}{2}\left(1+4x^2\right)^{3/2}(8x) \stackrel{!}{=} 0$

$$\Rightarrow x = 0$$

(b) We use the observation: $\dfrac{d}{ds} = \dfrac{dx}{ds}\cdot\dfrac{d}{dx}$

$$= \frac{dx}{dt}\cdot\frac{dt}{ds}\cdot\frac{d}{dx}$$

$$= \frac{\dot{x}}{v}\frac{d}{dx} = \frac{1}{\sqrt{1+4x^2}}\cdot\frac{d}{dx}$$ 2

$$\underline{e}_t = \frac{d}{ds}\underline{r} = \frac{1}{\sqrt{1+4x^2}}\frac{d}{dx}\left(x\,\underline{E}_x - x^2\,\underline{E}_y\right)$$ 2

$$= \frac{1}{\sqrt{1+4x^2}}\left(\underline{E}_x - 2x\,\underline{E}_y\right)$$

$$\underline{e}_n = \frac{d}{ds}\underline{e}_t = \frac{1}{\sqrt{1+4x^2}}\frac{d}{dx}\left(\frac{1}{\sqrt{1+4x^2}}\left(\underline{E}_x - 2x\,\underline{E}_y\right)\right)$$ 2

$$= \frac{1}{\sqrt{1+4x^2}}\left[-\frac{1}{2}\left(1+4x^2\right)^{-3/2}\cdot 8x\left(\underline{E}_x - 2x\,\underline{E}_y\right) + \frac{1}{\sqrt{1+4x^2}}\left(-2\,\underline{E}_y\right)\right]$$ product rule 1

$$= \frac{1}{\left(1+4x^2\right)^2}\left(-4x\,\underline{E}_x + \cancel{8x^2\,\underline{E}_y} - 2\,\underline{E}_y - \cancel{8x^2\,\underline{E}_y}\right)$$

$$= \frac{2}{\left(1+4x^2\right)^2}\left(-2x\,\underline{E}_x - \underline{E}_y\right)$$

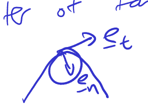
Need $\|\underline{e}_n\| = 1$, 1 and we note $1+4x^2 > 0$ 1

$$\Rightarrow \quad \underline{e}_n = \frac{1}{\sqrt{1+4x^2}}\left(-2x\,\underline{E}_x - \underline{E}_y\right)$$

Note if use $\underline{e}_t \cdot \underline{e}_n = 0$ — 1

then $\underline{e}_n = \pm\dfrac{1}{\sqrt{1+4x^2}}\left(2x\,\underline{E}_x + \underline{E}_y\right)$ — 1

To get sign: $\underline{e}_n$ must point to center of tangent circle

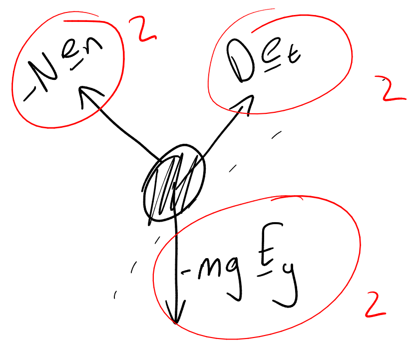


→ Need negative along $\underline{E}_y$ 2

$$\Rightarrow \underline{e}_n = -\frac{1}{\sqrt{1+4x^2}}\left[2x\,\underline{E}_x + \underline{E}_y\right]$$

Must explain choice of sign for full credit.

(c)(i) [6]



N: normal

D: drive/wheel force

mg: weight

Each force must have a clear direction

Must have a tangential force, can call it friction or anything else

D & N can have signs flipped.

(ii) $\underline{a} = \cancel{\frac{dv}{dt}}^{\,0} e_t + K v^2 e_n$

$\underline{F} = m\,\underline{a}$ (1)

$\underline{F}\cdot \underline{e}_n = m\,\underline{a}\cdot e_n \Rightarrow -N - mg\, e_n \cdot \underline{E}_y = mKv^2$ (2)

OK if N has opposite sign, as long as it's also flipped in FBD

$e_n \cdot \underline{E}_n = -\frac{1}{\sqrt{1+4x^2}}$ (1) $\Rightarrow N = +m\left(\frac{g}{\sqrt{1+4x^2}} - \frac{2v^2}{(1+4x^2)^{3/2}}\right)$

$$|N| = \frac{m}{\sqrt{1+4x^2}}\left| g - \frac{2v^2}{(1+4x^2)} \right|$$

(iii) [6] At a position x, let $v_c(x)$ be the speed so that $|N| = 0$

$|N| = 0 \Rightarrow g = \frac{2v_c(x)^2}{1+4x^2} \Rightarrow v_c(x) = \sqrt{\frac{g}{2}(1+4x^2)}$ (2)

$v_{c,min}$ is the lowest such speed

By inspection: $v_{c,min}$ occurs at $x^* = 0$ (min of $1+4x^2$) (2)

Or: $\frac{\partial v_c}{\partial x} \overset{!}{=} 0$: $2v_c \cdot \frac{\partial v_c}{\partial x} = \frac{g}{2}(8x) \overset{!}{=} 0$

$v_c > 0$

$\Rightarrow \frac{\partial v_c}{\partial x} = 0 \Rightarrow x = 0$ (2)

OR: $v_c = \sqrt{\frac{g}{2}(1+4x^2)}$

Comput $\frac{dv_c}{dx} \overset{!}{=} 0 \Rightarrow x^* = 0$ (2)

$v_{min} = v_c(x^*) = v_c(0) = \sqrt{g/2}$ (2)