Midterm examination 1

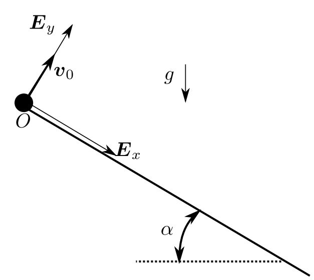
2 Questions for a total of 40 points in 40 minutes

Instructions:

- The exam is closed-book, closed-notes.
- No calculators allowed.
- Write only in pen, not pencil.
- Clearly number all solutions.
- We have no patience for dishonesty or apparent dishonesty.
- You may tear off the front page (these problems).
- Write your name / student ID on all solution sheets.

Question 1: A particle launched over a slope [10 points]

A particle of mass m starts at the origin at a velocity $\boldsymbol{v}_0 = v_{0y}\boldsymbol{E}_y$, as shown on the figure. The motion is over a slope, which makes an angle of α with the horizontal.



(a) [2 points] Draw a free-body diagram of the particle in flight. Neglect any aerodynamic drag effects, and clearly give the direction of all forces.

(b) [5 points] Show that the time at which the particle impacts the slope is given by

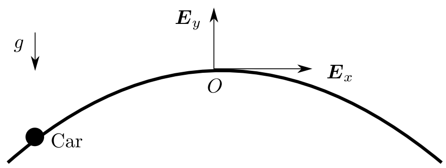
$$T = \frac{2v_{0y}}{g\cos\alpha}$$

(b) [3 points] How far from the origin along $\boldsymbol{E}_x$ does the particle impact the slope?

Please turn over

Question 2: A car drives over a peak [30 points]

A car of mass m drives over a parabolic peak at a constant speed v moving from left-to-right, shown in the figure below. Treat the car as a particle for this problem.



The car's position is given by $\boldsymbol{r} = x\boldsymbol{E}_x + y\boldsymbol{E}_y$, and the car starts at $x = x_0$. The height of the road surface at a distance x along $\boldsymbol{E}_x$ is given by $-x^2$.

(a) [**5 points**] Assume that the car remains on the surface of the road. Show that the rate with which the vehicle moves along the horizontal axis is as below, showing all steps.

$$\dot{x} = \frac{v}{\sqrt{1+4x^2}}. \tag{1}$$

At which point along the trajectory is the horizontal component of velocity maximized?

(b) [**9 points**] Assume that the car remains on the surface of the road. Recall the definition of the normal-tangential coordinates:

$$\boldsymbol{e}_t := \frac{\mathrm{d}}{\mathrm{d}s}\boldsymbol{r}$$
$$\kappa\boldsymbol{e}_n = \frac{\mathrm{d}}{\mathrm{d}s}\boldsymbol{e}_t$$

Starting with the definition show that the following hold:

$$\boldsymbol{e}_t = \frac{1}{\sqrt{1+4x^2}}\left(\boldsymbol{E}_x - 2x\boldsymbol{E}_y\right)$$
$$\boldsymbol{e}_n = \frac{1}{\sqrt{1+4x^2}}\left(-2x\boldsymbol{E}_x - \boldsymbol{E}_y\right)$$

Hint: Note that one can compute $\boldsymbol{e}_n$ *without the curvature* κ*. You may find it helpful to express* $\mathrm{d}s$ *in terms of* $\dot{x}$*,* v*, and* $\mathrm{d}x$*.*

(c) At a sufficiently high speed $v > v_c(x)$, the car will leave the surface of the road at horizontal position x. You may take for granted the curvature of the road κ, and the acceleration vector $\boldsymbol{a}$, as given below:

$$\kappa = \frac{2}{(1+4x^2)^{\frac{3}{2}}}$$
$$\boldsymbol{a} = \frac{\mathrm{d}v}{\mathrm{d}t}\boldsymbol{e}_t + \kappa v^2\boldsymbol{e}_n$$

Perform the following three steps, explicitly:

(i) [**6 points**] Draw a free body diagram of the car while it is on the road, clearly labelling all forces and indicating their directions.

(ii) [**4 points**] Show that the magnitude of the normal force that the road exerts on the car as a function of the speed v and the horizontal position x is given by

$$\frac{m}{\sqrt{1+4x^2}}\left|g - \frac{2v^2}{1+4x^2}\right|$$

(iii) [**6 points**] For each horizontal coordinate x along the road, there exists a critical speed v_c so that the normal force is zero. Derive the expression for v_c. What is the lowest speed, $v_{c,\min}$ for which the normal force is zero? Where does this occur along the curve?

Name: ____

Student ID: ____

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I certify that I will uphold the UC Berkeley Honor Code on this exam.

Signature ____

Name: ______________________________

Student ID: ______________________________

Name: ______________________________

Student ID: ______________________________

Name: ______________________________________

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