Midterm 2 (Yildiz) solutions

Problem 1

a) If the planet rotates faster, it will lose material from its surface at the equator.

b) Newton's second law for a particle of mass m on the equator is

$$mR\omega^2 = Gm\frac{M}{R^2} \tag{1}$$

where

$$M = \frac{4}{3}\pi R^3 \rho. \tag{2}$$

Solving for the period $T = 2\pi/\omega$ gives

$$T = 2\pi\sqrt{\frac{3}{4\pi G\rho}} \tag{3}$$

2a

(a) Let's just worry about the motion of the Center of Mass. This has mass $2m$ and accelerates from $v=0$ to $v=v$. The force F acts on it over a distance d_{cm}.

$$Fd_{cm} = \frac{1}{2}(2m)v^2 - \frac{1}{2}(2m)0^2$$

$$v = \sqrt{\frac{Fd_{cm}}{m}}$$

we want to eliminate d_{cm}. Use $d_{cm} = d - \frac{\ell}{2}$

$$v = \sqrt{\frac{F(d-\frac{\ell}{2})}{m}}$$

(b) The force is applied over a total distance d, so

$W = Fd$.

The kinetic Energy is $\frac{1}{2}(2m)v^2$

$$f = \frac{\frac{1}{2}(2m)v^2}{Fd} = \frac{m\frac{F(d-\frac{\ell}{2})}{m}}{Fd} = \frac{d-\ell/2}{d}.$$

Note that when $\ell \to 0$, $f \to 1$, so all the work becomes KE. When $\ell \to 2d$, $f \to 0$, so all the work is used to bring the masses together.

Problem 3

a) Each second another 5 kg of sand gains 1 m/s of velocity in the horizontal direction, so the rate of change of horizontal momentum is 5 N.

b) The force of friction is the only horizontal force acting on the sand, so by Newton's second law it has magnitude 5 N.

c) The conveyor belt moves at constant velocity, so the net force on it is zero. By Newton's third law, the sand exerts 5 N left, so the external force must exert 5 N to the right.

d) Since the force is constant, we just multiply by the displacement to get work: $W = Fd = Fvt = (5\text{ N})(1\text{ m/s})(1\text{ s}) = 5\text{ J}$.

e) The part of the sand that gains kinetic energy goes from 0 kinetic energy to $\frac{1}{2}mv^2 = \frac{1}{2}(5\text{kg})(1\text{m/s})^2 = 5/2\text{ J}$.

f) Some mechanical energy is lost, ultimately to heat, as the sand falls on the conveyor. This is not surprising if we think of the process as many small collisions, which are totally inelastic since the sand and the conveyor move with the same velocity afterwards.

Spring 2018 Yildiz Midterm # 2, Problem 4 Solution

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Rolling without slipping happens at the time T when $v(T) = R\omega(T)$. Therefore what we want to solve for is $v(t)$ and $\omega(t)$ with initial conditions being $\omega(t=0) = -\omega_0$ (where $\omega_0 > 0$) and $v(t=0) = v_0$. Because the wheel is initially rolling while slipping there is a kinetic friction force acting to the left at the base of the wheel, $F_{fr} = \mu_k N = \mu_k Mg$ as $N = Mg$ as the wheel is not accelerating up or down. Newtons second law then tells us that $Ma = -\mu_k Mg$ and therefore

$$v(t) = v_0 - \mu_k g t \tag{1}$$

To solve for $\omega(t)$ we need to calculate the net torque from the frictional force. Because we took the initial angular velocity to be negative, into the paper is the positive direction. Using the right hand rule we see that τ_{fr}'s direction is into the paper and therefore $\tau_{fr} > 0$. Because the $\vec{F}_{fr}$ is perpendicular to $\vec{r}$ we know $\tau_{fr} = R\mu_k Mg$. Therefore $\omega(t) = -\omega_0 + \frac{Mg\mu_k R}{I_{ball}}t$. $I_{ball} = \frac{2}{5}MR^2$ and therefore

$$\omega(t) = -\omega_0 + \frac{5g\mu_k}{2R}t \tag{2}$$

I. PART A

If the ball comes to a complete stop then this means there is a time T such that $v(T) = 0$ and $\omega(T) = 0$. Therefore

$$0 = v_0 - \mu_k g T \tag{3}$$

$$0 = -\omega_c + \frac{5g\mu_k}{2R}T \tag{4}$$

which is a system of two equations with two unknowns, T and ω_c. $T = \frac{v_0}{\mu_k g}$ and therefore

$$\boxed{\omega_c = \frac{5g\mu_k}{2R}\frac{v_0}{\mu_k g} = \frac{5v_0}{2R}} \tag{5}$$

II. PART B

In this part ω_0 is no longer and unknown and we just need to solve for the time T when $v(T) = R\omega(T)$.

$$v_0 - \mu_k g T = -\frac{\omega_c R}{2} + \frac{5\mu_k g}{2} T \tag{6}$$

$$\rightarrow \frac{9v_0}{4} = \frac{7}{2}\mu_k g T \tag{7}$$

Therefore

$$\boxed{T = \frac{9v_0}{14\mu_k g}} \tag{8}$$

and

$$\boxed{v(T) = v_0 - \frac{9v_0}{14} = \frac{5}{14}v_0} \tag{9}$$

III. PART C

This part is basically the same as part B except equation 6 changes to

$$v_0 - \mu_k g T = -2\omega_c R + \frac{5\mu_k g}{2} T \tag{10}$$

$$\rightarrow 6v_0 = \frac{7}{2}\mu_k g T \tag{11}$$

Therefore

$$\boxed{T = \frac{12v_0}{7\mu_k g}} \tag{12}$$

and

$$\boxed{v(T) = v_0 - \frac{12}{7}v_0 = -\frac{5}{7}v_0} \tag{13}$$

5.(a) Energy conservation:

$$\frac{1}{2}kd^2 + mg\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Rolling without slipping:

$$v = R\omega.$$

Disk/cylinder:

$$I = \frac{1}{2}MR^2.$$

Solving for ω:

$$\omega = \sqrt{\frac{2kd^2 + 4mgd\sin\theta}{(M+2m)R^2}}.$$

(b) Work done by kinetic friction:

$$\mu_k mgd\cos\theta.$$

Word done by frictional torque (analogous to Fd:

$$\tau\Delta\theta = \tau_{fr}\frac{d}{R}.$$

Energy conservation with energy lost:

$$\frac{1}{2}kd^2 + mgd\sin\theta = \frac{1}{2}mR^2\omega^2 + \frac{1}{4}MR^2\omega^2 + \mu_k mgd\cos\theta + \tau_{fr}\frac{d}{R}.$$

Solving for ω:

$$\omega = \sqrt{\frac{2kd^2 - 4\tau_{fr}d/R + 4mgd\sin\theta - \mu_k mgd\cos\theta}{(M+2m)R^2}}.$$

(c) Complete stop occurs at some equilibrium displacement. By balance of forces along the incline:

$$mg\sin\theta = kd_{eq} \implies d_{eq} = \frac{mg}{k}\sin\theta.$$

By total energy conservation:

$$\frac{1}{2}kd^2 + mgd\sin\theta = \frac{1}{2}kd_{eq}^2 - mgd_{eq}\sin\theta + \mu_k mgx\cos\theta + \tau_{fr}\frac{x}{R}.$$

Solving for x:

$$x = \frac{\frac{1}{2}k(d^2 - d_{eq}^2) + mg(d+d_{eq})\sin\theta}{\mu_k mg\cos\theta + \tau_{fr}/R}.$$