Physics 7B - Lecture 3 - Midterm 1 - Problem 1 - Solution

For each part, there was only one correct answer. **3 points** were awarded for selecting the correct and only the correct answer. **1 point** was awarded for selecting both the correct answer and one incorrect one. No points were given otherwise.

a) Option **c.** is correct. The pressure an ideal gas exerts on it's surroundings is due to the change in momentum in the molecules as they hit a wall. It is there even in the absence of gravity (so a. is false), and would exist even if the molecules did not interact with each other, as long as they interacted with the wall (so b. is false). The pressure also does not rely on the exact distribution of velocities (so d. is false), and will be there as long as not all molecules have zero velocity.

b) Since the gas is ideal, we can apply $PV = nRT$. We are given that the pressure is halved, while the volute is doubled. Therefore, the left-hand side of the ideal gas law remains constant, implying the right-hand side must remain constant as well. Given that n is held constant, we conclude that RT has not changed as the gas goes from initial to final state.Thus, the internal energy, which (by equipartition) is $\frac{1}{2}nRT$ per available degree of freedom, also remains constant, i.e., the correct answer is **a.**

c) The freezing of degrees of freedom at low temperatures is due to the existence of discrete energy levels as predicted by quantum mechanics (option **c.** is correct). It has nothing to do with our ability (or lack thereof) to perform a measurement or with collisions (options a. and d. are incorrect). It leads to a reduction of heat capacities, but is not caused by it (option b. is incorrect).

d) The correct answer is **d.** Since no energy is exchanged between the gas and its surroundings, its total energy (and thus average energy) remains unchanged. Thus, the temperature is constant. Note that this also means that the distribution of speeds is unchanged. The increase in volume leads to both longer average time between collisions with the walls, and greater wall area, both of which lead to a decrease in pressure. Even though no heat is exchanged, this process leads to an increase in entropy. This is possible because $dS = \frac{dQ}{T}$ only for reversible processes, and the removal of the barrier is irreversible. Entropy is a state variable, and it's change can be computed by $\Delta S = \int \frac{dQ}{T}$ for a reversible process between the same initial and final state. Alternatively, from the microscopic description of entropy (as proportional to the log of the number of microstates corresponding to a given macrostate), it is also clear that the entropy increases.

   The gas is not in equilibrium as it expands, but we were given no information to allow us to conclude anything about whether it is ideal (part e. is incorrect).

Q2.

a) Q/t = KA (100)/L
   Direction: Hotter end to colder end.

b) Q/t = mL, Therefore, m = Q/t*L (Mass/Time)

c) Thermal Conductivity of Metal (k) is greater than wood. Hence, wood would be worse.

Midterm 1: Problem 3

Problem statement

One mole of an ideal monatomic gas is taken from an initial state (P_i, V_i) to a final state $(P_f = 3P_i, V_f = 3V_i)$.

Find the change in entropy, ΔS, in terms of P_i, V_i, and relevant physical constants for cases (a) and (b) below. Calculate the entropy change explicitly in both cases.

- **(a)** The gas first expands isothermally until its volume is tripled and then increases pressure at constant volume until it reaches its final pressure.
- **(b)** The gas first compresses isothermally until its pressure is tripled and then expands at constant pressure until it reaches its final volume.
- **(c)** Plot these paths on a P-V diagram. Label each part of the manipulation clearly.

Solution

(a) For a reversible process, the relationship between heat and entropy is given by

$$\mathrm{d}S = \frac{\mathrm{d}Q}{T}.$$

We will assume that all processes in this problem are reversible.

The first process to consider in case (a) is the isothermal expansion from $V_i \to 3V_i$. The temperature during this expansion is constant, so the change in entropy is

$$\Delta S_1 = \frac{1}{T_i}\int \mathrm{d}Q \quad = \frac{Q}{T_i},$$

where Q is the total amount of heat added. From the first law of thermodynamics and the equipartition theorem,

$$\Delta U = Q - W, \quad \text{and} \quad \Delta U = \frac{d}{2}nR\Delta T,$$

we see that $\Delta T = 0 \implies Q = W$. The work done during this expansion is found to be

$$W = \int P dV = nRT_i \int_{V_i}^{3V_i} \frac{\mathrm{d}V}{V} = nRT_i \ln 3.$$

The gas does positive work, so it absorbs positive heat: $Q > 0$. The change in entropy is also positive:

$$\Delta S_1 = nR\ln 3.$$

Lastly, we note that $n = 1$:

$$\Delta S_1 = R\ln 3.$$

Next, we consider the isovolumetric process. Because the temperature now varies, the change in entropy is

$$\Delta S_2 = \int \frac{\mathrm{d}Q}{T}.$$

Here we can use the relationship

$$\mathrm{d}Q = nC_V\mathrm{d}T,$$

where

$$C_V = \frac{d}{2}R.$$

Setting $n = 1$ and $d = 3$ (as is the case for a monatomic ideal gas), we get

$$\Delta S_2 = \frac{3}{2}R\int_{T_i}^{T_f}\frac{\mathrm{d}T}{T} = \frac{3}{2}R\ln\left(\frac{T_f}{T_i}\right).$$

The ideal gas law tells us that the temperature is proportional to the product PV; therefore $T_f = 9T_i$, and

$$\Delta S_2 = \frac{3}{2}R\ln 9 = 3R\ln 3.$$

Finally, the total change in entropy for case (a) is

$$\Delta S = \Delta S_1 + \Delta S_2 = 4R\ln 3.$$

(b) Because entropy is a state variable and cases (a) and (b) involve the same initial and final states, the value we calculate for ΔS in part (b) must equal our result from part (a).

During the isothermal compression, the change in entropy is again

$$\Delta S_1 = \frac{W}{T_i}.$$

This time work is being done *on* the gas, so $W < 0$ (as is the change in entropy). The pressure increases by a factor of 3 during this compression; so by the ideal gas law, the volume must decrease by a factor of 3:

$$\Delta S_1 = \frac{1}{T_i}\int P\mathrm{d}V = R\int_{V_i}^{V_i/3}\frac{\mathrm{d}V}{V} = R\ln\left(\frac{1}{3}\right) = -R\ln 3$$

(where we've already set $n = 1$).

During the isobaric expansion, we can use the relationship

$$\mathrm{d}Q = C_P\mathrm{d}T,$$

where

$$C_P = C_v + R = \frac{5}{2}R.$$

So

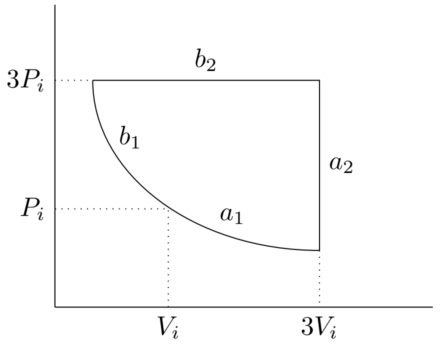
$$\Delta S_2 = \int\frac{\mathrm{d}Q}{T} = \frac{5}{2}R\int_{T_i}^{T_f}\frac{\mathrm{d}T}{T} = \frac{5}{2}R\ln 9 = 5R\ln 3.$$

The total change in entropy is

$$\Delta S = \Delta S_1 + \Delta S_2 = 4R\ln 3,$$

as it must be.

(c) A rough P-V diagram (not to scale) is shown below. Path $a = a_1 + a_2$ is taken counterclockwise from the initial point to the final point, and path $b = b_1 + b_2$ is taken clockwise.



Physics 7B, Lecture 3, MT1

Problem 4 Solution

Part a

We are asked to express γ as a ratio of integers. In general, $\gamma = \frac{C_P}{C_V}$, and both C_P and C_V depend on the number of degrees of freedom of the gas (d): $C_V = \frac{d}{2}$ and $C_P = C_V + R$. For a diatomic gas, there are 3 translational degrees of freedom, 2 rotational degrees of freedom, and some vibrational degrees of freedom (ignored). So here we have

$$d = 3 + 2 = 5.$$

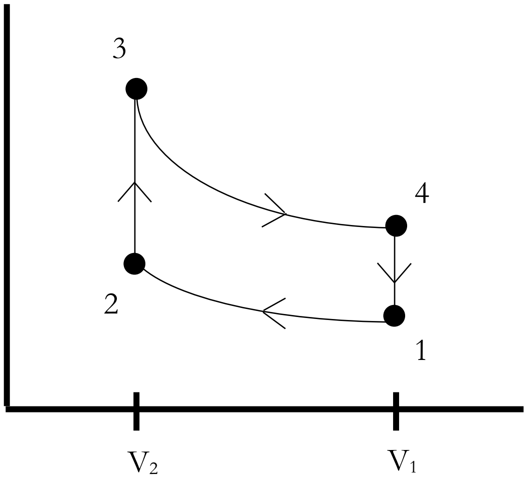
Therefore,

$$C_V = \frac{5}{2}R, \quad C_P = \frac{5}{2}R + R = \frac{7}{2}R.$$

So, calculating the ratio of the molar specific heats gives us

$$\gamma = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \mathbf{\frac{7}{5}}.$$

Part b



Part c

We can see right away that $Q_1 = Q_3 = 0$ because both these processes are adiabatic. We see also that $W_2 = W_4 = 0$ because there is no change in volume during these steps. Also, because there is no heat transfer during processes 1 and 3, the change in entropy is zero for both these processes.

W_1 and W_3 can be calculated using the 1st law of thermodynamics: $\Delta E_{int} = Q - W$. We know that $\Delta E_{int} = \frac{d}{2} nR\Delta T$, so we can relate the work to the change in temperature and then rewrite our answer in terms of pressure and volume (our known quantities) using the ideal gas law. For process 1, $Q_1 = 0$, so

$$\Delta E_{int} = \frac{5}{2} nR\Delta T = Q_1 - W_1 = -W_1.$$

Plugging in temperatures yields

$$-W_1 = \frac{5}{2} nR(\mathrm{T}_2 - T_1).$$

Rewriting in terms of pressure and volume using $PV = nRT$ and solving for work gives us

$$W_1 = -\frac{5}{2}(P_2V_2 - P_1V_1).$$

An analogous calculation for process 3 shows that

$$W_3 = -\frac{5}{2}(P_4V_4 - P_3V_3) = \frac{5}{2}(P_3V_2 - P_4V_1).$$

Notice that we have rewritten our answer in terms of our known volumes. To find heat in processes 2 and 4 we can again use the 1st law of thermodynamics, but this time work is zero and heat is nonzero. For process 2,

$$\Delta E_{int} = Q_2 - W_2 = Q_2 - 0 = \frac{5}{2} nR\Delta T.$$

From this we can write the heat in terms of the temperature change, which we can rewrite in terms of pressure and volume as we did previously for the work.

$$Q_2 = \frac{5}{2} nR(T_3 - T_2) = \frac{5}{2}(P_3V_3 - P_2V_2) = \frac{5}{2}V_2(P_3 - P_2).$$

An analogous calculation for process 4 gives us

$$Q_4 = \frac{5}{2}(V_1P_1 - V_4P_4) = \frac{5}{2}V_1(P_1 - P_4).$$

To calculate the change in entropy over steps 2 and 4, we must integrate because temperature is not constant. We note that the heat involved in these steps can be calculated using the molar specific heat at constant volume because these are isovolumetric processes. So we need to evaluate

$$\Delta S = \int \frac{dQ}{T} = \int \frac{nC_V}{T}\,dT.$$

For process 2, this integral reads

$$\Delta S_2 = \frac{5}{2}nR\int_{T_2}^{T_3}\frac{1}{T}\,dT = \frac{5}{2}nR\ln\left(\frac{T_3}{T_2}\right).$$

We need to write the ratio of temperatures in terms of known quantities, so we must use the ideal gas law to replace the temperatures with pressures and volumes. Doing so yields

$$\Delta S_2 = \frac{5}{2}nR\ln\left(\frac{\frac{P_3V_3}{nR}}{\frac{P_2V_2}{nR}}\right) = \frac{5}{2}nR\ln\left(\frac{P_3}{P_2}\right).$$

Notice that we have canceled the volumes because the process between points 2 and 3 is isovolumetric. We can find the change in entropy for step 4 by observing that the total change in entropy must add up to zero or by completing an analogous calculation. This shows us that

$$\Delta S_4 = \frac{5}{2}nR\ln\left(\frac{P_1}{P_4}\right) = \frac{5}{2}nR\ln\left(\frac{P_2}{P_3}\right).$$

In summary, our results for the table are:

	W	Q	ΔS
Process 1 (point 1 to point 2)	$\frac{5}{2}(P_1V_1 - P_2V_2)$	0	0
Process 2 (2 to 3)	0	$\frac{5}{2}V_2(P_3 - V_2)$	$\frac{5}{2}nR\ln\left(\frac{P_3}{P_2}\right)$
Process 3 (3 to 4)	$\frac{5}{2}(P_3V_2 - P_4V_1)$	0	0
Process 4 (4 to 1)	0	$\frac{5}{2}V_1(P_1 - P_4)$	$\frac{5}{2}nR\ln\left(\frac{P_2}{P_3}\right)$

Each box was worth 1 point. Note that $P_4 = \frac{P_1P_3}{P_2} = P_3\left(\frac{V_2}{V_1}\right)^{\gamma}$. Solutions written using these relationships were also considered correct.

Part d

The efficiency of a heat engine in general is given by

$$e = 1 - \frac{Q_L}{Q_H}.$$

In this case, Q_L refers to the heat in process 4 and Q_H is that of process 2. (You can determine this by seeing which heat is positive and which is negative, which you can see by looking at the differences in pressure. The positive heat is Q_H and the negative one is Q_L.) One should be able to calculate the efficiency by using the results from the previous part or by writing out the heats in terms of the molar specific heat and changes in temperature. We will take the latter approach here:

$$e = 1 - \frac{nC_V(T_4 - T_1)}{nC_V(T_3 - T_2)}.$$

We want to rewrite this expression in terms of γ, which shows up in the adiabatic pressure-volume relationship:

$$PV^\gamma = contant.$$

Our expression for the efficiency is written in terms of temperature, so we will need to combine this relationship with the ideal gas law, which connects pressure and temperature. Solving the ideal gas law for pressure gives

$$P = \frac{nRT}{V},$$

which can be inserted into the adiabatic relationship as follows

$$\left(\frac{nRT}{V}\right)V^\gamma = constant.$$

The number of moles and the gas constant are both constant, so we can absorb them into the righthand side. This yields a new adiabatic relationship in terms of temperature and volume:

$$TV^{\gamma-1} = new\ constant.$$

We can apply this relationship to processes 1 and 3, which yields

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1},$$

$$T_3V_3^{\gamma-1} = T_4V_4^{\gamma-1}.$$

We can then stick these relationships into our expression for the efficiency to eliminate two of our temperatures (here we choose to eliminate T_1 and T_4):

$$e = 1 - \frac{T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} - T_2\left(\frac{V_2}{V1}\right)^{\gamma-1}}{T_3 - T_2}.$$

We notice that $V_3 = V_2$ and $V_4 = V_1$, so $\frac{V_3}{V_4} = \frac{V_2}{V_1}$. This simplifies our expression as follows:

$$e = 1 - \frac{\left(\frac{V_2}{V_1}\right)^{\gamma-1}(T_3 - T_2)}{(T_3 - T_2)} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}.$$

We are almost there. Now we just need to replace the ratio $\frac{V_1}{V_2}$ with r. Doing so and cleaning up our answer a bit yields

$$e = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - r^{1-\gamma}.$$

Part e

Reversible processes are quasistatic (performed very slowly so that they are a series of equilibrium states) and involve no nonconservative forces. The performance of this cycle too quickly to be quasistatic or with friction forces, as would be the case in a real engine, would make the process irreversible.

Problem 5_Wurtele's Exam

a) $\frac{dQ}{dt} = \in \sigma\,(4\pi R^2)(T^4 - \ 3^4)\ where\ T\ varies$
b) $dQ = \ -mcdT\quad Temperature\ goes\ down\ in\ the\ process$
c) $\frac{dQ}{dt} = \in \sigma\,(4\pi R^2)(T^4)\quad After\ neglecting\ the\ temperature\ of\ environment = 3K\ \approx tends\ to\ 0$

$$-mc\,\frac{dT}{dt} = \in \sigma\,(4\pi R^2)(T^4)$$

$$\frac{-mc}{\in \sigma\,(4\pi R^2)}\int_{330}^{300}\frac{dT}{T^4} = \int_0^{\tau} dt$$

$$\frac{-mc}{\in \sigma\,(4\pi R^2)}\left[\frac{T^{-4+1}}{-4+1}\right]_{330}^{300} = \ \tau$$

$$\frac{mc}{3 \in \sigma\,(4\pi R^2)}\left[\frac{1}{T^3}\right]_{330}^{300} = \ \tau$$

$$\frac{mc}{3 \in \sigma\,(4\pi R^2)}\left[\frac{1}{300^3} - \frac{1}{330^3}\right] = \ \tau\ \ Remember\ time\ should\ be\ positive$$

d) Increases
The rate at which spherical ball radiates heat decreases (dQ/dt goes down) and time to cool increases.